Abstract — The neuro-synaptic system is a molecular communication network which is important in the human body. In this system, the neurons and junctions between them, i.e., synapses, are used for the transmission. The axonal transmission and the synaptic diffusion cause interference to the adjacent neurons. We model this effect with the interference channel studied in the communication theory. Then, we propose detection and equalization methods to compensate the effects of the interference and improve the signal detection probability at the destination neurons. Finally, we evaluate the performance of the proposed schemes with simulations. The simulations show that in an interference neuronal channel, the proposed equalization methods outperform the non-equalized system.

I. INTRODUCTION

Nano-molecular-machines are the building blocks of future nanotechnology devices. They contain sets of arranged molecules, which perform very simple computations, sensing and actuating tasks. Because of their limited processing ability, there is a need to interconnect them to form a network and build up complex machines [1]. Molecules are information carriers in the molecular communication. Among all the molecular communication systems in the human body, the neuro-synaptic network plays a vital role in the nervous system. This network develops a communication path between the brain and other organs to carry the neural information all around the body using electrical impulses and neurotransmitters [2].

The neuro-synaptic network uses neurons and the synaptic cleft between them as its communication channel. A neuron has different parts where each part is responsible for a specific job in the nervous system. For example, the soma generates spikes, dendrites receive neurotransmitters from other neurons and axons propagate the spikes. Each of these roles can be modeled as a specific function in the communication system. A descriptive illustration of a neuron cell’s components can be found in [4]. The diffusion of neurotransmitters in synapses is modeled with molecular communications [3]. The data transmission through the axons takes place with the electrical spikes and can be modeled with traditional wired systems. The multiple sclerosis (MS) is an illness in the body nervous system which damages the myelin sheaths in the axons of the neurons [5]. This leads to some disabilities like inability to walk and visual impairments [7].

The authors in [8] have developed a multiuser model for the neuro-synaptic system and investigated the overall capacity of the communicational link. In [9], the analysis of the performance of the multi-input single-output (MISO) synaptic channel has been provided. In [10] and [11], a system with the inter-symbol interference of neurons is studied. A neuronal channel with interference between neurons has not been considered in the previous works, and this is the main motivation of our work.

In this paper, we propose a multiuser interference channel model for the neuro-synaptic communication, which models the effect of the inter-neuron interference. We model the interference strength in the axonal transmission with a new coefficient. We also consider the effect of the axonal noise in our model. Then, we apply the central and the distributed multiuser detection and equalization methods to improve the performance of the receiver in the destination neurons. We also evaluate the proposed methods by simulations to show the improvements in the performance of the signal detection. The simulations show that these methods outperform the non-equalized neuronal system. These techniques can be used both for the therapeutic solutions in the central nervous system (CNS) or performance advancement in the artificial nano-networks. In artificial nano-networks, the communication between neurons is theoretically modeled [1]. They may be used for the future treatment of disorders.

II. SYSTEM MODEL

We consider the multiuser interference channel model for our neural system. As you can see in Fig. 1, there are \( L \) input and \( L \) output neurons in this network. Here, the interaction between the neurons consists of two sources, one with the axonal interference and the other with the diffusional interference. As mentioned earlier, the MS damages the myelin sheaths. This causes a leakage in the \( Na^{2+} \) flux through the axonal transmission resulting in interference to nearby axons. On the other hand, the neurotransmitter diffusion in the multiple access media is a random process leading neurotransmitters to disperse toward different destinations. Thus, this random molecular diffusion takes part in the overall multiple access synaptic network. Before introducing the interference channel model in detail, we focus on the point-to-point channel shown with dashes in Fig. 1.

A. Point-to-Point Channel

The point-to-point channel model is a basic interference-free building block of this structure. This model is shown in Fig. 2 dashed part, which is similar to the model used in [12]. The input for this model is a random stimulus from the body nervous system which is in the electrical signal form. This stimulus enters the linear-nonlinear-Poisson (LNP) encoding block which generates the spike train with Poisson stochastic process property [9]. The spike trains are in the form of \( S(t) = \sum_{k} \delta(t - t_k) \), where \( \delta(.) \) is the Dirac delta function and \( t_k \)'s are the incidence times of the spikes.

In order to study the properties of this system, it is convenient to quantize the time into small bins, each with duration \( \tau \) [13]. Regarding the refractory property of the neurons, the value of \( \tau \) is accordingly chosen providing that there is at most
one spike in each bin [14]. Thus, the information at time slot \(k\) would be \(I_k = 1\) when there is a spike at the \(k\)th time slot and \(I_k = 0\) for the case of the spike absence.

Similar to [3], we model the axonal noise which is caused by random opening in ionic channels [15] with a binary process of probability \(P_o\). This noise occurs along the axon tubes and we indicate it with axonal noise block in Fig. 2. In this block, “1” represents a spike and “0” stands for no incoming spike.

Vesicles are bags of neurotransmitters which play an important role in carrying the signals. Once a spike arrives at the synapse, based on the number of the vesicles (which determines the channel conditions), there is a release probability in vesicles, \(P_r\). Similar to [12], we model this process with a Z channel model called the vesicle release in Fig. 2. In this block, “1” in the output represents a release and “0” stands for no vesicle release. After this release, the neurotransmitters spread toward the postsynaptic receptors in the receiving neuron. This diffusion process causes a random time delay in the receiving signal. For simplicity, we assume that the receiver can estimate this time delay. There is a trial-to-trial variability in the amplitude of the postsynaptic responses observed from the receiving neuron, which incorporates a variable gain \(q\) to the amplitude of the signal. These quantal amplitudes fit the Gamma distribution [4].

Next, in postsynaptic terminals, there is an excitatory post synaptic potential (EPSP) waveform shape function which generates an electrical signal responding to the received neurotransmitters. This function is modeled with an alpha – function [4], in the form of \(h(t) = \frac{1}{\tau_e} \exp (1 - \frac{t}{\tau_e})u(t)\), where \(u(t)\) is the step function. The value of \(\tau_e\) is related to the type of the receptors in the postsynaptic area [3]. Therefore, the corresponding signal is in the form of \(w(t) = \sum_k B_k q_k h(t - t_k)\).

Finally, the synaptical noise which is caused by the release of the vesicles and ionic fluxes at the postsynaptic terminal is modeled by a white Gaussian noise \(n(t)\) with zero mean and spectral density of \(N_0/2\) [13]. Thus, the output of this point-to-point channel is in the following form

\[
r(t) = w(t) + n(t) = \sum_k B_k q_k h(t - t_k) + n(t),
\]

where subscript \(k\) represents the \(k\)th time slot and \(B_k\) is a binary random variable which shows whether the received neurotransmitters can initiate an action potential or not. In other words, \(B_k = 1\) if an action potential is generated due to the received neurotransmitters. Otherwise, it is set to zero, i.e., \(B_k = 0\). Also, \(q_k\) is the value of random gain \(q\) in the \(k\)th time slot. A verification of the above mentioned functions using experimental data can be found in [6].

B. Multi-Neuron Channel Model

The hypothetical model for the synaptical multiuser interference channel has been illustrated in Fig. 2. After the EPSP wave shaping block, the multiuser interference channel block models the multiple access feature of the nervous system. Let the signals to the inputs of the multiuser interference block be \(w(t) = [w_1(t) \ w_2(t) \ \cdots \ w_L(t)]^T\), the synaptic noise be \(n(t) = [n_1(t) \ n_2(t) \ \cdots \ n_L(t)]^T\), and finally the output of the block be \(y(t) = [y_1(t) \ y_2(t) \ \cdots \ y_L(t)]^T\). The relation between the components of \(w(t)\) and \(y(t)\) can be described as \(y_i = g_{i1}w_1(t) + \sum_{j \neq i} g_{ij}w_j(t) + n_i(t)\). Hence, we denote it by the following matrix equation

\[
y(t) = Gw(t) + n(t),
\]

where \(G\) is a matrix of \(g_{ij}\)s which denote the mutual interference between the \(j\)th input, denoted by \(w_j(t)\) and the \(i\)th output denoted by \(y_i(t)\). This interference weight depends on the physiological characteristics of the neuron and the synapse which is the result of the synaptic and the axonal interferences. Analogous to the synaptic mutuality in [8], we extend their definition to the mutual interference weight as the following

\[
g_{ij} = \sqrt{E_c} \beta_j(w_{ij} + v_{ij}),
\]

where \(\sqrt{E_c}\) represents the power of the input signal \(w_{ij}(t)\), \(\beta_j\) is a variable taking the value of either +1 or −1 to represent whether the presynaptic terminal is excitatory or inhibitory, respectively. The value of \(u_{ij}\) shows the interference strength in the axonal transmission between \(w_{ij}(t)\) and \(w_{ij}(t)\), which depends on their distance from each other in the nano-network. Finally, \(v_{ij}\) is the synaptical interference weight between the input terminal \(i\) and the output terminal \(j\). Furthermore, due to sensitivity of the receptors [8], the channel state information (CSI) is obtained at the receiver side. This means that the receiver knows the elements of \(G\). The mechanism for getting CSI at the receiver side is fully described in [8] and [10].

After the multiuser interference channel, there are \(L\) different filters, each matched to one of EPSP shaping functions in the sending neurons. The reason of using the matched filter is to maximize the SNR at the output [16].

The next block in Fig. 2 is the equalizer which is placed to reduce the effect of the interference. In the rest of the paper, we discuss the different methods for designing the equalizer and compare performances of them in various conditions. The reduction in the interference improves the signal transmission reliability in the nano-network.

Finally, the detector block makes decisions based on some decision rules to complete the communication system procedure. More details about this block are discussed in the next sections. Although we view the system of Fig. 2 as an interference channel, we use all the outputs to make a decision. Thus, we deal with an interference channel with a centralized decision system.

III. Equalizer Design to Avoid Multiuser Interference

In this section, we describe some methods to design an equalizer to help improve the performance of a multi-neuron channel with inter-neuron interference. We study these systems in the following:

A. Maximum Likelihood Multiuser Detector

Here, we derive the solution of the Maximum Likelihood (ML) detector, which maximizes the conditional probability density of the received signal given the transmitted message. In order to find the solution, we define \(\Gamma(t) \triangleq \)
Fig. 2. Hypothetical model for the synaptic multiuser interference channel.

\[ \sum_{i=1}^{L} A_i x_i(t) h^{(i)}(t) + \sigma_L n(t), \text{ where } A_i = \sum_{k=1}^{K} g_{ki} \text{ and } \sigma_L^2 = \sum_{i=1}^{L} \text{Var}[n_i(t)] \text{ and } x_i, \text{'s are the inputs for EPSP block.} \]

Here, the problem is to optimally detect the vector \( \mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_L(t)]^T \) at the receiving neuron. According to [17], the most likely \( \mathbf{x}(t) \) maximizes

\[ \Omega(\mathbf{x}(t)) = 2 \int_0^T \sum_{i=1}^{L} A_i x_i(t) h^{(i)}(t) \Gamma(t) \] \[ - \int_0^T \sum_{i=1}^{L} A_i x_i(t) h^{(i)}(t) \, dt \] \[ = 2 \sigma^2 \mathbf{A} \hat{\mathbf{y}} - \mathbf{x}^T \mathbf{H} \mathbf{x}, \]

\[ \text{(4)} \]

where \( \hat{\mathbf{y}}(t) = \left[ \hat{y}_1(t), \hat{y}_2(t), \ldots, \hat{y}_L(t) \right]^T \) is the vector of matched filter outputs, i.e. \( \hat{y}_i(t) = \int_0^T \Gamma(t) h^{(i)}(t) \, dt \).

Moreover, \( \mathbf{A} \) is an \( L \times L \) diagonal matrix, with elements \( A_{ii} = \text{diag}[A_1, \ldots, A_L] \) and the unnormalized cross correlation matrix is \( \mathbf{H} = \mathbf{ARA} \), where \( \mathbf{R} \) is the normalized cross correlation matrix with diagonal elements equal to 1 and non-diagonal elements are \( r_{ij} = \int_0^T h^{(i)}(t) h^{(j)}(t) \, dt, \, i \neq j \).

The expression in (4) is a combinatorial optimization problem (COP) [17]. Unlike the other optimization problems, these kinds of problems can be always solved by an exhaustive search. Due to this complexity of the multiuser detector (MUD), it is desirable to use methods that have less complexity in the implementation.

B. Other Multiuser Detectors

In this section, we introduce some MUD methods that exhibit good performance-complexity trade-offs. We assume that all the neurons in the system are identical and have the same response time. Thus, when a stimulus is applied to this network, all of the neurons act synchronously. Therefore, they have the same timing information which is known for every neuron in the multiuser interference channel.

1) The Decorrelating Detector (DD): The output vector of the matched filter system shown in Fig. 2 is given by the following

\[ \hat{\mathbf{y}}(t) = \mathbf{RA} \mathbf{x}(t) + \hat{\mathbf{n}}(t), \]

\[ \text{(5)} \]

where \( \hat{\mathbf{n}}(t) \) is a Gaussian noise vector with zero mean and covariance matrix \( \sigma^2 \mathbf{R} \), where we assume that all elements of \( \mathbf{n}(t) \) have variances equal to \( \sigma^2 \). First, we define \( \mathbf{Q} \equiv \mathbf{RA} \). In order to design a simple neuronal receiver, which does not need the knowledge of the received amplitudes, we just premultiply the vector of matched filter outputs by \( \mathbf{Q}^{-1} \).

Thus, we have

\[ \mathbf{Q}^{-1} \hat{\mathbf{y}}(t) = \mathbf{Q}^{-1} \mathbf{Q} \mathbf{x}(t) + \mathbf{Q}^{-1} \hat{\mathbf{n}}(t) = \mathbf{x}(t) + \mathbf{Q}^{-1} \hat{\mathbf{n}}(t), \]

\[ \text{(6)} \]

It is worth mentioning that the components of (6) are all interference-free and the only source of randomness is the background synaptic noise. This method is called "decorrelating detector in the centralized model". We can also perform this method using the distributed computation mode. It is more convenient for our system model to have separate neurons acting independently. To see this method, we use \( Q^{-1}_{ij} \) as a shorthand for \( (Q^{-1})_{ij} \), and also define

\[ \tilde{h}^{(i)}(t) = \sum_{j=1}^{L} Q^{-1}_{ij} h^{(j)}(t). \]

\[ \text{(7)} \]

We can use \( \tilde{h}^{(i)}(t) \) s as orthogonal bases for the decorrelating detector. In other words, instead of using \( h^{(i)}(t) \) as a matched filter for each neuron, we use \( \tilde{h}^{(i)}(-t) \). This method is analogous to zero-forcing equalization in inter symbol interference (ISI) [16]. Thus, we expect similar properties for both of them, like the noise enhancing property which reduces the performance of the system in high SNR regimes.

2) Minimum Mean Square Error (MMSE) Multiuser Detector: An important approach in the linear detector design is the problem of minimizing the mean square error between our observation and estimation. Thus, we need to observe all the outputs of the neurons in the centralized model and also we assume that we have knowledge about the amplitudes in the source neurons. In this part, we assume that we want to detect the vector \( \mathbf{x}(t) \), and then, by applying detection rules in the point-to-point section, we can recover nervous information vector \( \mathbf{S}(t) \). Thus, the minimization problem has the following structure

\[ \min_{c_i} E[(x_i - \langle c_i, \hat{y} \rangle)^2]. \]

\[ \text{(8)} \]

where the MMSE detector chooses the waveform \( c_i \) for each neuron to achieve the subject in (8).

We can formulate the problem in (8), into a matrix form with uncoupled optimization problems [17]. By this, we can solve all problems simultaneously by choosing the \( L \times L \) matrix \( \mathbf{M} \), that achieves

\[ \min_{\mathbf{M} \in \mathbb{R}^{L \times L}} E[\|\mathbf{x} - \tilde{\mathbf{y}}\|^2]. \]

\[ \text{(9)} \]

Note that the expectation in (9) is with respect to \( \mathbf{x}(t) \). We know that \( \|\Lambda\|^2 = \text{trace}(\Lambda \Gamma) \). First, it is better to obtain the covariance matrix of the error vector

\[ \text{cov} \{x - \tilde{y}\} = (\mathbf{x} - \tilde{\mathbf{y}})(\mathbf{x} - \tilde{\mathbf{y}})^T \]

\[ = E[x x^T] - E[x \tilde{y}^T] M^T M E[\tilde{y} \tilde{y}^T] + M E[\tilde{y}^T] M^T. \]

\[ \text{(10)} \]

In addition, using the fact that the noise and the data are uncorrelated, we simplify (10) as the following

\[ \text{cov} \{x - \tilde{y}\} = E_0 (\mathbf{I} + \gamma \mathbf{ARA})^{-1} E_0 (\mathbf{M} - \tilde{M})(\mathbf{RA}^2 \mathbf{R} + \gamma^{-1} \mathbf{R})(\mathbf{M} - \tilde{M})^T \]

\[ \text{(11)} \]

where \( \tilde{M} \equiv A^{-1}(\mathbf{R} + \gamma^{-1} \mathbf{A}^{-2})^{-1} \) and \( E_0 \) is the energy of the nervous pulses, \( x(t) \). Furthermore, we substitute the SNR value, \( \frac{E_0}{\gamma} \), with \( \gamma \). On the other hand, we form the matrix \( \mathbf{A} \) only for active neurons in the system, thus, we assure that \( \mathbf{A} \) is nonsingular. Therefore, we rewrite the problem in (9) as the following

\[ \min_{\mathbf{M} \in \mathbb{R}^{L \times L}} E[\|\mathbf{x} - \tilde{\mathbf{y}}\|^2] = \min_{\mathbf{M} \in \mathbb{R}^{L \times L}} \text{trace} \{\text{cov} \{x - \tilde{y}\}\}. \]

\[ \text{(12)} \]

Considering the nonnegative definite manner of the matrix \( \mathbf{RA}^2 \mathbf{R} + \gamma^{-1} \mathbf{R} \), we conclude that the trace of the second term in (11) is always nonnegative. Thus, the minimization problem reduces to the following

\[ \min_{\mathbf{M} \in \mathbb{R}^{L \times L}} E[\|\mathbf{x} - \tilde{\mathbf{y}}\|^2] = \text{trace} \{[\mathbf{I} + \gamma \mathbf{ARA}]^{-1}\}. \]

\[ \text{(13)} \]

Here, we can implement the solution in (13) by multiplying the \( (\mathbf{R} + \gamma^{-1} \mathbf{A}^{-2})^{-1} \) matrix to the vector of the matched filter outputs.
3) Successive Interference C canceller (SIC) Multiuser Detector: The idea behind this method is based on the multistage decision. This means that after detecting one of the neurons in one stage, we can subtract its impact. Therefore, the effect of the multi-neuron interference is reduced at each stage. It is worth mentioning that if the decision about the detected neuron is not correct, this error propagates in the next steps. Therefore, we assume that the decision at each step is correct. It is obvious that the order of detecting neuron signals plays an important role in the performance of the system. At the detection stage, we order the neurons according to their SINR values. We use the matched filter outputs to estimate the above quantity.

For the case of \(2 \times 2\) neural multiuser interference system, suppose that we first detect neuron 1 and show its detected data with \(\hat{x}_1\). Thus, after reconstructing its transmitting signal, we have
\[
\hat{\Gamma}(t) = \Gamma(t) - A_1\hat{x}_1h_1(t) = A_2x_2h_2(t) + A_1(x_1 - \hat{x}_1)h_1(t) + n(t).
\]

Now, by processing \(\hat{\Gamma}(t)\) with the filter matched to neuron 2, we have\(\hat{\Gamma}(t) \ast h_2(-t) = A_2x_2 + A_1(x_1 - \hat{x}_1)\rho_{12} + \sigma_{12}n(t)\). We see that the effect of the decision error for neuron 1 is \(x_1 - \hat{x}_1\) term. With a complete interference cancellation at the first stage, i.e., \(x_1 = \hat{x}_1\), we have a simpler detector for neuron 2, that only faces the background noise. Extending this model for a system with \(L\) neurons is straightforward. After sorting the signal strengths and when making a decision about the neuron \(l\), we assume that the decisions for neurons \(l+1, \ldots, L\) are correct and we also neglect the effect of neurons \(1, \ldots, l-1\). Thus, we make our decision based on \(\hat{y}_l = \sum_{i=l+1}^{L} A_i\rho_{i,l}\hat{x}_i\).

IV. Simulation Results

In this section, we analyze the performance of our proposed detection systems and compare the introduced methods with each other. We also use Monte-Carlo method with \(10^7\) generated bits to simulate the communication channel using the vesicle-based approach.

For the multiuser interference channel, first we compare the detection methods with each other in a \(2 \times 2\) network with identical neurons each with a spike generation probability of 0.5. To focus on the multiuser detection part of the system, we assume that the quantal amplitude is fixed. For the first neuron, we use \(\tau_1 = 0.1\) and for the other one we have \(\tau_2 = 1\). We also use \(g_1 = g_2 = 1\) and \(g_{12} = g_{21} = 0.5\) for \(G\).

Fig. 3 shows the comparison between the proposed techniques and also with the no-equalization case. As it can be observed, MSE and SIC Equalizers have very similar performances and both of them are better than the DD technique. This is because of the noise enhancing property of the DD, where in higher SINRs, the noise becomes stronger and affects the performance of the detector. Furthermore, all the detection methods outperform the plain system and we can conclude that our proposed schemes could be applied as a possible MS treatment.

In Fig. 4, we compare the error probability of different multiuser interference networks with unequal number of neurons. We use the \(\tau_1 = [\tau_1, \tau_2, \tau_3, \tau_4] = [0.1, 0.4, 1.2, 3]\) vector as the time constants of the neurons and also we apply an equiprobable condition in spike generation for every neuron. According to the rate bound mentioned in [9], with a constant data rate, as the number of neurons increases in the multiuser interference network, the performance of the detection system degrades.

REFERENCES