Jamming Game for Secure OFDMA Systems

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Abstract—In a communication system, security and reliability have an important impact on the quality of services. In this paper, we investigate power and subcarrier allocation of a transmitter over the downlink of an OFDMA system with security considerations among users in the presence of a jammer. This paper considers the interaction between the jammer and the transmitter as a zero-sum game where the objective function is the transmitter’s secrecy sum-rate. In order to allocate available resources, we solve the optimization problem of each player to characterize their optimal strategies. Considering the intractability of the closed form Nash equilibrium for general power regime, we show the existence of pure Nash equilibrium point for low total power regimes of the transmitter and the jammer and attain an expression for each regime. Finally, we examine the jamming power effect on the secrecy sum-rate and acquire lower and upper bounds of this rate using simulations.

Index Terms—Physical Layer Security, Zero-Sum Game, OFDMA, Resource Allocation

I. INTRODUCTION

In a wireless communication system due to the broadcasting and superposition properties, not only we should reduce the interference in order to reach a high reliable system, but also the security of the communication should also be considered. In [1], Shannon introduced his seminal work on secrecy systems in which he established the perfect secrecy concepts. Next important work on the security was done by Wyner in [2] in which he introduced the wiretap channel and secrecy capacity concepts. He showed that in order to achieve a perfect secure communication, the transmitter must send data in a rate lower than the secrecy capacity. Csiszar and Kroner investigated the security of broadcast channel with two users in [3] and showed that we can find channel codes to have a reliable and secure transmission. The mentioned works established the bases of the physical layer security. With increasing use of wireless devices and demands for higher rates, the OFDM technique as a multi-carrier system obtained many applications in wireless networks. In [4], resource allocation for multi-carrier broadcast channel by considering optimizing achievable secrecy rate was investigated. In [5], dynamic resource allocation for broadband wireless channels which uses OFDMA was studied. In this paper, users were divided into two groups: users with security constraints, and normal users that do not need secure channel. Optimization problem was formulated and solved under a total transmit power constraint for the base station while maintaining an average secrecy rate for each individual secure user. In [6], power allocation and artificial noise design are applied to confuse the eavesdropper, and also in [7], resource allocation problem was studied on OFDMA networks that use decode-and-forward relays to generate artificial noise.

In addition to dealing with the eavesdroppers, we should handle the presence of jammers as a threatening factor for wireless systems reliability. In [8], jamming problem formulation on a wiretap parallel Gaussian channel was investigated. In this paper, there is a jammer besides an eavesdropper. The authors in [10] use a game-theoretic framework to study competitive interactions between a jammer, a secondary user, and a transmitter-receiver pair. This paper considers restrictions on knowledge of channel gains, power budget, and physical interference. In [11], authors investigate the power allocation problem for cognitive radio where they assume the existence of a smart jammer. In [12], the authors examine the optimal power allocation strategies for OFDM wiretap channels. They formulate a zero-sum power allocation game and consider the secrecy rate as a payoff function.

In this paper, we consider the interaction between a transmitter and a jammer as a zero-sum game, assuming that the transmission rate is the utility function. Indeed, we suppose that the jammer has adversarial effects on the transmission. We assume that the system is a downlink multi-user OFDMA with security consideration such that the users eavesdrop each other. The equilibrium point of this two-player zero-sum game is equivalent to the optimal resource allocation of both players, simultaneously. We reach this goal by solving the optimization problem of each player, separately which needs to solve the joint optimal power and subcarrier allocation. Because this game is continuous, the equilibrium point is hard to reach. Thus, we obtain a closed form solution for two low power regime scenarios, namely insignificant total power of jammer and insignificant total power for transmitter. Finally, we examine the analytical results using simulation realizations.

II. SYSTEM MODEL

We assume a communication system where a transmitter wants to send confidential data to $K$ users. The transmitter uses OFDMA technique with $N$ subcarriers. It is considered that for each individual user, the other $K - 1$ users act as eavesdroppers. In addition, there is a jammer that is aware of the system and aims to disturb data transmission with jamming signal. Transmitter-receiver and jammer-receiver channels are slow-fading. With aforementioned assumptions, optimal power and subcarrier allocation among users to maximize secrecy
sum-rate is the transmitter’s objective. In contrast, the jammer aims to minimize secrecy sum-rate. Therefore the interaction between transmitter-receiver and the jammer is a zero-sum game. We assume that the power used in the transmitter and the jammer is constrained to a total power. Also channel state information (CSI) of all channels is known by the transmitter and the jammer. Also, we assume that the jammer utilizes directive antennas.

We can obtain the received signal at the main channel $m$ of user $i$ as

$$Y_{mi} = A_{mi}X_{Ti} + Z_{mi}, \quad i = 1, 2, ..., K,$$  \hspace{1cm} (1)

where $A_{mi}$ is a diagonal matrix of channel coefficients between the transmitter and $i$th user, $X_{Ti}$ is a $N \times 1$ vector of transmitted signal, $Z_{mi}$ is additive Gaussian noise, $A_{mi}$ is diagonal matrix of channel coefficients between jammer and $i$th user, and $X_{Jmi}$ is a $N \times 1$ vector which denotes jamming signal.

The received signal at the eavesdropper side of $i$th user can be written as

$$Y_{ei} = A_{ei}X_{T} + Z_{ei},$$  \hspace{1cm} (2)

in which $A_{ei}$ is a $N \times N$ matrix of channel coefficients between transmitter and eavesdropper when sending signals to user $i$, $X_{T}$ is a $N \times 1$ vector of transmitted signal to all users, and $Z_{ei}$ is a $N \times 1$ vector denoting channel’s additive Gaussian noise. Because of considered directive antennas for the jammer, there is no jamming signal at the eavesdropper.

The achievable secrecy rate for the $i$th user over the $j$th subcarrier can be calculated as [5]

$$r_{ij}^s = \log \left(1 + \frac{|A_{ij}p_{si,j}|^2}{\sigma_j^2} + \frac{\alpha_{ij}^2p_{si,j}}{\sigma_j^2} + \frac{\zeta_{ij,m}p_{J,m,i,j}}{\sigma_j^2}\right)^+,$$  \hspace{1cm} (3)

in which $p_{si,j}$ and $p_{J,m,i,j}$ represent allocated transmitter’s power to user $i$ on subcarrier $j$ and allocated jammer’s power, respectively and $\sigma_j^2$ indicates noise average power. $\alpha_{ij} = |A_{mi,j}|^2$ and $\zeta_{ij,m} = |A_{m,j}|^2$ denote the channel power gains of the transmitter and the jammer when subcarrier $j$ is allocated to user $i$, respectively. Also, $\beta_{ij} = \max_{k,j} \alpha_{kj}$ states that among all wiretap channels a channel with best condition determines the achievable secrecy rate. It means positive achievable secrecy rate on each subcarrier is restricted to two best channel coefficients and the user with the best channel condition has the only positive secrecy rate [5]. With this assumption subcarrier allocation is easily known for each given channel condition. Now assume that $\Theta (\gamma) = \{\Theta_i(\gamma)\}_{i=1}^K$ be the set of allocated subcarriers to the users in which $\Theta_i(\gamma)$ demonstrates the set of allocated subcarriers to the user $i$ and $\gamma$ is the set of the channels’ parameters for each channel realization. $\Theta_i(\gamma)$’s satisfy following conditions

$$\Theta_1(\gamma) \cup \Theta_2(\gamma) \cup ... \cup \Theta_K(\gamma) \subseteq \{1, 2, ..., N\},$$  \hspace{1cm} (4a)

$$\Theta_{i_1}(\gamma) \cap \Theta_{i_2}(\gamma) = \emptyset \quad \forall \ i_1 \neq i_2 \in K.$$  \hspace{1cm} (4b)

Therefore, secrecy sum-rate of user $i$ is defined as follow

$$r_i^s = \sum_{j \in \Theta_i} r_{ij}^s = \sum_{j \in \Theta_i} \left[ \log \left(1 + \frac{\alpha_{ij}p_{si,j}}{\sigma_j^2} + \frac{\zeta_{ij,m}p_{J,m,i,j}}{\sigma_j^2}\right)^+ \right] - \log \left(1 + \frac{\beta_{ij}p_{si,j}}{\sigma_j^2}\right)^+.$$

The receiver uses weighted sum rate as an objective function for power and subcarrier allocation. In this case the achievable secrecy rate at the transmitter’s side will be

$$C^*(P_T, P_J; \Theta) = \sum_{i=1}^K \sum_{j \in \Theta_i} w_i \left[ \log \left(1 + \frac{\alpha_{ij}p_{si,j}}{\sigma_j^2} + \frac{\zeta_{ij,m}p_{J,m,i,j}}{\sigma_j^2}\right)^+ \right] - \log \left(1 + \frac{\beta_{ij}p_{si,j}}{\sigma_j^2}\right)^+,$$  \hspace{1cm} (6)

where $w_i$ is the weighted coefficient of user $i$ and depends on the QoS requirements, $P_T$ and $P_J$ are defined as follows

$$P_T = (p_{T1}, p_{T2}, ..., p_{TK})^t,$$  \hspace{1cm} (7a)

$$P_J = (p_{J1}, p_{J2}, ..., p_{JK})^t,$$  \hspace{1cm} (7c)

$$P_{J} = (p_{J1}, p_{J2}, ..., p_{JK})^t.$$  \hspace{1cm} (7d)

### III. ZERO-SUM TRANSMITTER-JAMMER GAME FORMULATION

Now, according to this fact that transmitter’s goal is to maximize secrecy sum-rate using power and subcarrier allocation and jammer’s aim is to minimize it, thus interaction between these entities can be considered as zero-sum game. Objective function in this game is transmitter’s secrecy sum-rate. Therefore, objective function can be considered as (6).

We suppose the following power constraints for the transmitter,

$$p_{T,i,j} \geq 0, \quad \forall i, j,$$  \hspace{1cm} (8a)

$$\sum_{i=1}^K \sum_{j \in \Theta_i} p_{T,i,j} \leq P_{Ttotal},$$  \hspace{1cm} (8b)

where $P_{Ttotal}$ is the total available power of transmitter and (8a) and (8b) are non-negative power and total power constraints, respectively. We also assume that the jammer’s power constraints are as

$$p_{J,i,j} \geq 0, \quad \forall i, j,$$  \hspace{1cm} (9a)

$$\sum_{i=1}^K \sum_{j \in \Theta_i} p_{J,i,j} \leq P_{Jtotal},$$  \hspace{1cm} (9b)

in which $P_{Jtotal}$ is the jammer’s total power.

Considering above mentioned assumption about objective function and power constraints, we can introduce zero-sum game with objective function as (6) and power constraints as (8) and (9).

**Proposition 1:** Consider the zero-sum between the transmitter and the jammer as (6). If we define minmax =
\[
\min \max_{\psi(\gamma)} C^{*}(P_{T}, P_{J}; \Theta), \text{ and } \max \min_{\psi(\gamma)} C^{*}(P_{T}, P_{J}; \Theta)
\]
and the set of jammer’s pure strategies as
\[
\psi(\gamma) = \left\{ p_{J,i,j} \in \mathbb{R}^{+} ; i = 1, ..., K; j = 1, ..., N : \sum_{i,j} p_{J,i,j} \leq P_{\text{Total}} \right\}
\]
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\psi(\gamma) = \left\{ p_{J,i,j} \in \mathbb{R}^{+} ; i = 1, ..., K; j = 1, ..., N : \sum_{i,j} p_{J,i,j} \leq P_{\text{Total}} \right\}
\]
As you can see, these two sets are closed and convex. Thus, considering the subcarrier allocation as the set \( \Theta \) and the real objective function \( C^{*}(P_{T}, P_{J}; \text{boldsymbol(\Theta)}) \), we use theorem 2.2 [9] and conclude that
\[
\min \max_{\psi(\gamma)} C^{*}(P_{T}, P_{J}; \Theta) \geq \max \min_{\psi(\gamma)} C^{*}(P_{T}, P_{J}; \Theta)
\]

A. Transmitter’s optimization problem

According to the fact that the transmitter’s goal is to maximize the secrecy sum-rate, using (6) as objective function which is the secrecy sum-rate, and (8a) and (8b) as constraints, the optimization problem of the transmitter will be
\[
\max_{P_{T}} C^{*}(P_{T}, P_{J}; \Theta)
\]
\[
\text{s.t. } p_{T,i,j} \geq 0, \forall i, j,
\]

B. Jammer’s optimization problem

Considering the jammer’s goal of minimizing the secrecy sum-rate denoted by (6) and its constraints as (9a) and (9b), we can write the jammer’s optimization problem as
\[
\min_{P_{J}} C^{*}(P_{T}, P_{J}; \Theta)
\]
\[
\text{s.t. } \sum_{i,j \in \Theta_{i}} p_{J,i,j} \leq P_{\text{Total}}.
\]

IV. OPTIMAL TRANSMITTER RESOURCE ALLOCATION

In this section, considering the optimization problem of the transmitter at the previous section, we obtain the optimum power allocation. According to the problem and considering the set \( \Theta \) and using Carollary 2 [4] which yields to the condition \( \alpha_{ij} \geq \beta_{ij} + \frac{\zeta_{m,i,j} p_{m,i,j}}{\sigma_{ij}^{2}} \) \forall i, j, the objective function is concave and the power constraint is convex with respect to \( p_{T,i,j} \). Thus, if we use the duality method to solve the resource allocation problem, the problem reaches to optimal solution, asymptotically. Assuming that \( \mathcal{P}(\gamma) \) be the set of all the non-negative power elements \( \{p_{T,i,j}\} \) for every determined channel condition \( \gamma \) such that for every subcarrier \( j \) there is only one positive \( p_{T,i,j} \). Therefore, Lagrange dual function will be
\[
L(\mu, \lambda) = \max_{\{p_{T,i,j} \in \mathcal{P}(\gamma)\}} \left\{ \sum_{i=1}^{K} \sum_{j=1}^{N} r_{ij}^{s}(p_{T,i,j}, \gamma) + \sum_{i=1}^{K} \sum_{j=1}^{N} \mu_{j} p_{T,i,j} + \lambda \left( P_{T} - \sum_{i=1}^{K} \sum_{j=1}^{N} p_{T,i,j} \right) \right\}
\]
where the expression \( r_{ij}^{s}(p_{T,i,j}, \gamma) \) determines the achievable secrecy rate of user \( i \) at the subcarrier \( j \) based on the power allocation within the set \( \mathcal{P}(\gamma) \). In addition, \( \mu = [\mu_{ij}]_{i \in K, j \in N} \geq 0 \) and \( \lambda \geq 0 \) are the Lagrange coefficients for the transmitter’s power constraints. Thus, we have the dual problem as
\[
\min_{\mathcal{L}} L(\mu, \lambda)
\]
\[
s.t. \mu \geq 0, \lambda \geq 0.
\]

According to (14), we could see that the dual function can be decomposed to \( N \) sub-function as follows
\[
L(\mu, \lambda) = \sum_{j=1}^{N} L_{j}(\mu, \lambda) = \sum_{j=1}^{N} \mathcal{R}_{j}(\mu, \lambda, \{p_{T,i,j}\})
\]
\[
\mathcal{R}_{j}(\mu, \lambda, \{p_{T,i,j}\}) = \sum_{i=1}^{K} \left( \mu_{j} p_{T,i,j} \right) - \lambda \left( p_{T} - \sum_{i=1}^{K} p_{T,i,j} \right)
\]

A. Optimality condition for power and subcarrier allocation and optimal power extraction

Considering that function \( \mathcal{R}_{j}(\mu, \lambda, \{p_{T,i,j}\}) \) is a concave function of \( p_{T,i,j} \), we can find its maximum applying KKT conditions. In this case we assume that subcarrier \( j \) is allocated to user \( i \). So our goal is to allocate power to this subcarrier. Thus, we have the following KKT conditions,
\[
P_{T} \geq 0, \sum_{i,j} p_{T,i,j} \leq P_{\text{Total}}, \lambda \geq 0, \mu \geq 0
\]
\[
\mu_{ij} p_{T,i,j} = 0, \forall i, j.
\]
\[
\frac{\partial \mathcal{R}_{j}(\mu, \lambda, \{p_{T,i,j}\})}{\partial p_{T,i,j}} = \frac{w_{i} \alpha_{ij}}{\left( \sigma_{ij}^{2} + \zeta_{m,i,j} p_{m,i,j} + p_{T,i,j} \alpha_{ij} \right)} - \frac{w_{i} \beta_{ij}}{\left( \sigma_{ij}^{2} + \zeta_{m,i,j} p_{m,i,j} + p_{T,i,j} \beta_{ij} \right)} = 0.
\]
According to (17b), if \( p_{T,i,j} \geq 0 \), it implies that \( \mu_{ij} = 0 \). With some simplifications on (17c), we can find out the following equation,
\[
\frac{w_{i} \left( \alpha_{ij} \left( \sigma_{ij}^{2} + \zeta_{m,i,j} p_{m,i,j} + p_{T,i,j} \alpha_{ij} \right) - \beta_{ij} A_{ij} \right)}{A_{ij} \left( \sigma_{ij}^{2} + \zeta_{m,i,j} p_{m,i,j} + p_{T,i,j} \beta_{ij} \right)} = \lambda - \mu_{ij}
\]
in which \( A = \left( \sigma_{ij}^{2} + \zeta_{m,i,j} p_{m,i,j} + p_{T,i,j} \alpha_{ij} \right) \). Thus by noting that \( \lambda \geq 0 \), the condition that a positive power value is
allocated for user $i$ on subcarrier $j$ is obtained as the following condition is satisfied,

$$\alpha_{ij} \geq \beta_{ij} + \frac{\zeta_{m,ij} P_{Jm,ij}}{\sigma_{ij}^2}. \quad (19)$$

This condition shows that for every subcarrier $j$ a positive power value is allocated for user $i$, if the channel power gain, i.e. $\alpha_{ij}$ for this user has the maximum amount among all users. This condition provides a subcarrier allocation strategy for all users. With this condition, subcarriers are allocated and the only thing that needs to be done is to optimize power allocation.

To obtain optimum power allocation we solve (17c) with assuming $\mu_{ij} = 0$, therefore the optimal power can be attained as

$$p_{T,i,j}^* = \frac{a}{2a_{ij}b_{ij}^2} \left[ 1 + \frac{4a_{ij}b_{ij}}{a_b^2} \left( \frac{\alpha_{ij}^2 - \beta_{ij} c w_i - b}{\lambda} - 1 \right) \right], \quad (20)$$

in which $a = \alpha_{ij}^2 + \beta \left( \sigma_{ij}^2 + \zeta_{m,ij} P_{Jm,ij} \right)$, $b = \sigma_{ij}^2 \left( \sigma_{ij}^2 + \zeta_{m,ij} P_{Jm,ij} \right)$ and $c = \left( \sigma_{ij}^2 + \zeta_{m,ij} P_{Jm,ij} \right)$.

In addition to (19), positive power condition is obtained as follows

$$\alpha_{ij} \geq \beta_{ij} + \frac{\zeta_{m,ij} P_{Jm,ij}}{\sigma_{ij}^2} + \frac{\lambda \left( \sigma_{ij}^2 + \zeta_{m,ij} P_{Jm,ij} \right)}{\lambda}, \quad (21)$$

Thus, the optimal power allocation will be as shown in (22) at the top of the following page. Since subcarriers’ allocation and optimal powers have been calculated, now we can update dual parameters using inserting powers in (16) to obtain optimal solution.

B. Dual parameters update

To update and obtain optimal values of $\lambda$ and $\mu$, we substitute the calculated powers in order to calculate $L (\mu, \lambda)$. Also, it should be noticed that based on (17b) updating of $\mu_i$’s has no effect on optimal solution and $p_{T,i,j}^*$. To update $\lambda$, if the dual problem be always convex and differentiable, we can use gradient descent algorithm. But due to the discontinuity at subcarrier allocation, $L (\mu, \lambda)$ is not differentiable and this algorithm is not applicable. In this case instead of using gradient, sub-gradient function is applied to update dual parameters. For this problem, sub-gradient of $q (\lambda) = L (\mu, \lambda)$ is defined as

$$\partial q (\lambda) = P_{Total} - \sum_{i=1}^{K} \sum_{j=1}^{N} p_{T,i,j}^*. \quad (23)$$

For sub-gradient algorithm updating, if it is assumed that the algorithm is at the $k$th step of updating i.e., $\lambda^{(k)}$, then $\lambda$ at the step $(k+1)$ could be obtained as follows

$$\lambda^{(k+1)} = \lambda^{(k)} + \omega^{(k)} \partial q \left( \lambda^{(k)} \right), \quad (24)$$

in which $\omega^{(k)}$ is step length and defined as

$$\omega^{(k)} = \left( c + \frac{c}{\sqrt{k}} \right), \quad (25)$$

where $c$ is a constant.

C. Optimum power and subcarrier allocation for insignificant total power $P_{T, total} \to 0$

In this case, we use Taylor expansion. Solving this problem using Lagrange method leads to

$$\lambda = w_i \left( \frac{\alpha_{ij}}{\sigma_{ij}^2 + \zeta_{m,ij} P_{Jm,ij}} - \frac{\beta_{ij}}{\sigma_{ij}^2} \right), \quad (26)$$

Note that $\lambda$ depends on $i$ and $j$. So, after replacing $\lambda$ in dual function, the maximum value obtains only for one case which is

$$p_{T,i,j}^* = \begin{cases} p_r & \text{if } (i,j)=(t,k), \\ 0 & \text{if } (i,j) \neq (t,k). \end{cases} \quad (27)$$

This means that the entire power is allocated to only one user on one subcarrier.

V. JAMMER’S OPTIMUM RESOURCE ALLOCATION

In this section, we solve the jammer’s optimization problem presented (13). According to this problem and secrecy sum-rate equation, if there exist no power constraint on jammer’s power then the allocated jamming power to user $i$ on subcarrier $j$ satisfies the following condition

$$p_{Jm,i,j} \geq \frac{\sigma_{ij}^2 (\alpha_{ij} - \beta_{ij})}{\zeta_{m,ij} \beta_{ij}}, \quad (28)$$

otherwise, achievable secrecy rate would be zero. However, the jammer could make the achievable secrecy rate zero when it has enough power to jam all subcarriers for all users. Of course, if the jammer has enough power which satisfies (28) for all $i$ and $j$, then it chooses its strategies as

$$p_{Jm,i,j} = \begin{cases} \frac{\sigma_{ij}^2 (\alpha_{ij} - \beta_{ij})}{\zeta_{m,ij} \beta_{ij}} & \text{if } \alpha_{ij} \geq \beta_{ij}, \\ 0 & \text{if } \alpha_{ij} \leq \beta_{ij}. \end{cases} \quad (29)$$

Indeed, according to above equation, jammer allocates positive power when $\alpha_{ij} \geq \beta_{ij}$.

A. Optimal jammer’s power allocation for constrained power

In this section, we solve jammer’s optimization problem (13) with constrained power. First, we define set $\mathcal{F} (\gamma)$ like $\mathcal{P}(\gamma)$ as a set of all non-negative allocated powers $\{p_{Jm,j}\}$ for every given channel realization. With this definition, both constraints of allocated positive power and subcarrier is satisfied. According to definition of $\alpha_{ij}$ and $\beta_{ij}$, subcarrier allocation is fixed for a given channel condition. Since jammer allocates power to a subcarrier when achievable secrecy rate of user $i$ on subcarrier $j$ is positive, i.e. $\gamma^{(k)}_{ij} (p_{T,i,j}, p_{Jm,j}) \geq 0$, then subcarrier allocation policy will be $\alpha_{ij} \geq \beta_{ij}$ for every $j$. 

\[
\sum_{i=1}^{K} \sum_{j=1}^{N} p_{Jm,i,j} \geq \frac{\sigma_{ij}^2 (\alpha_{ij} - \beta_{ij})}{\zeta_{m,ij} \beta_{ij}}, \quad (28)
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\[
\sum_{i=1}^{K} \sum_{j=1}^{N} p_{Jm,i,j} \geq \frac{\sigma_{ij}^2 (\alpha_{ij} - \beta_{ij})}{\zeta_{m,ij} \beta_{ij}}, \quad (28)
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\]

Indeed, according to above equation, jammer allocates positive power when $\alpha_{ij} \geq \beta_{ij}$.
Consequently, we can write positive jamming power allocation condition as

$$\alpha_{ij} \geq \beta_{ij}. \quad (30)$$

This condition was obtained in the case that we don’t have power constraint (28). Now, we can form set of allocated subcarriers $$\Theta$$ using investigating $$\alpha_{ij} \geq \beta_{ij}$$ for every $$j$$. With determining $$\Theta$$ and considering jammer’s optimization problem, the objective function is convex related to $$p_{T_{i,j}}$$.

Thus, this problem is a convex optimization problem and its dual solution gives optimal solution without duality gap.

By following similar procedures as Subsection IV-A, optimal jamming power is obtained as (31) at the top of the page. According to this equation, we can see that if $$\lambda \leq \max_{i,j} \frac{w_i^2\alpha_{ij}\beta_{ij}^2P_{J_{i,j}}}{b}$$, no power will be allocated.

Actually, in this case it is not necessary to allocate any power because achievable secrecy rate is zero. If $$\lambda \leq \min_{i,j} \frac{w_i^2\alpha_{ij}\beta_{ij}^2P_{J_{i,j}}}{(1-\sigma_{ij}^2)\beta_{ij}^2P_{i,j}}$$, power allocation will be according to no power constraint case.

**B. Power and subcarrier allocation for insignificant total power**

Considering power constraint \( \sum_{i=1}^{K} \sum_{j=1}^{N} p_{J_{i,j}} \leq P_{J_{Total}} \), and following similar steps in Section IV-C we will have

$$p_{J_{i,j}} = \begin{cases} P_{J_{Total}} & \text{if } (i,j) = (t,k), \\ 0 & \text{if } (i,j) \neq (t,k), \end{cases} \quad (32a)$$

$$t,k = \arg \max_{i,j} \frac{w_i\alpha_{ij}\beta_{ij}P_{J_{i,j}}}{\sigma_{ij}^2(\sigma_{ij}^2 + \alpha_{ij}P_{i,j})}. \quad (32b)$$

Above equations state that the total power is allocated to one user on a single subcarrier.

**VI. SOLVING THE EQUILIBRIUM POINT OF THE GAME BETWEEN TRANSMITTER AND JAMMER**

According to zero-sum game stated in Section III, in this section we analyze zero-sum game and solve Nash equilibrium for low power constraint. We consider two cases to solve equilibrium point. In case I, we suppose that total power of the transmitter is insignificant, i.e. $$P_{Total} \rightarrow 0$$. Case II is related to the situation that the jammer’s total power is low, i.e. $$P_{J_{Total}} \rightarrow 0$$. For the first case, following proposition describes equilibrium point of the game.

**Proposition 2:** In zero-sum game between the transmitter and the jammer, in the case that total power of the transmitter is slight, i.e. $$P_{Total} \rightarrow 0$$, pure strategy Nash equilibrium will be as follows

$$\begin{align*}
& (p_{T_{i,j}}^*, P_{J_{i,j}}^*) = \left\{ \begin{array}{ll}
P_{J_{Total}}, & \text{if } (i,j) = (t,k), \\
0, & \text{if } (i,j) \neq (t,k),
\end{array} \right.
\end{align*} \quad (33)$$

in which index $$(t,k)$$ is obtained as follows

$$t,k = \arg \max_{i,j} \frac{w_i\alpha_{ij}\beta_{ij}P_{J_{i,j}}}{\sigma_{ij}^2(\sigma_{ij}^2 + \alpha_{ij}P_{i,j})} - \frac{\beta_{ij}}{\sigma_{ij}^2}. \quad (34)$$

**Proof:** According to calculated equations for optimal power in (27a) and (27b), if the total power of the transmitter is small, it allocates all its power to one user on one subcarrier. Thus, for a given channel realization, the jammer also jams the same subcarrier according to its perfect CSI.

For the second case, following proposition describes equilibrium point of the game.

**Proposition 3:** If the total power of the jammer is insignificant, i.e. $$P_{J_{Total}} \rightarrow 0$$, pure strategy Nash equilibrium point will be as (35) at the top of the following page in which index $$k$$ is obtained from the following equation

$$t,k = \arg \min_{i,j} \left( \max_{P_{i,j}} \frac{C_{ij}}{P_{J_{Total}}} \right) \quad (36)$$

Note that index $$t$$ in (35) is not important and it is only used for determining jamming power.

**Proof:** Based on equations (32a) and (32b), jammer uses all of its power to jam one subcarrier. Thus, at this subcarrier, optimal power allocation of transmitter is computed based on (22), but in the other subcarriers it is obtained from this equation in which $$p_{J_{i,j}}$$ is zero. Applying obtained equations to (32) leads to Nash equilibrium point.
The achievable secrecy rate for given jamming power, respectively. Indeed, these two curves specify upper and lower bounds on the transmitter’s power is distributed uniformly, respectively. No jamming and the case of jammers minimization in which best and worst achievable secrecy rates have been achieved for $P_T$ powers shown versus the transmitters power for two different jamming cases.

In Fig. 1, achievable secrecy sum-rate curves have been obtained pure Nash equilibrium points considering two low power cases for the transmitter and the jammer. Furthermore, we formed a zero-sum game between the jammer and the transmitter in presence of an active jammer. We solved the power and subcarrier allocation optimization problems of the transmitter and the jammer. Furthermore, we obtained pure Nash equilibrium points considering two low power cases for the transmitter and the jammer.

Fig. 2 has been obtained from solving minimization problem of jammer versus its power for two different transmitters power $P_{Total} = 0$, 10 dB. It shows jamming power effect on achievable secrecy rate. We can observe that by increasing jamming power achievable secrecy rate diminishes. It is because of the fact that the high the jammers power is, the worse the channel condition gets.

VIII. Conclusion

In this paper, we investigated the security of a multi-user downlink OFDMA system in presence of an active jammer. We formed a zero-sum game between the jammer and the transmitter with the secrecy sum-rate as the objective function. We solved the power and subcarrier allocation optimization problems of the transmitter and the jammer. Furthermore, we obtained pure Nash equilibrium points considering two low power cases for the transmitter and the jammer.

References