Dynamic Learning for Distributed Power Control in Underlaid Cognitive Radio Networks

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Abstract—In this paper, a distributed, minimum overhead power control algorithm for underlay cognitive radio networks (CRNs) having multiple primary and secondary users is proposed. The problem is formulated as a noncooperative game and a learning algorithm is proposed for optimizing the power allocation of secondary users. In the considered network, secondary users (SUs) do not have full information on the interference and power control strategies of other SUs. As a result, they update their strategy using a simple feedback from the primary user base station that provides the total interference. Although there is no cooperation among secondary users, it is shown that, under incomplete information, the proposed learning algorithm converges to the strategy of the players in the Nash equilibrium of the complete information case. The Nash equilibrium point is analytically derived and it is shown that, despite the fact that each user individually tries to maximize its own payoff, at the end, the proposed algorithm will converge to the complete information game Nash equilibrium point. It is also demonstrated that because of the slotted time assumption of the algorithm, it will be capable of adapting to a time-varying environment if some conditions on the SUs’ processing power are satisfied. Simulation results are then used to corroborate the analytical derivations.

I. INTRODUCTION

Spectrum efficiency is a challenging issue in today’s wireless communication systems. Cognitive radios (CRs) were proposed to solve this problem, in which the licensed primary users (PUs) allow the unlicensed secondary users (SUs) to exploit their vacant spectrum [1]–[3]. There are three methods for spectrum sharing in cognitive radio networks, called interweave, underlay, and overlay techniques [4]. Interweave systems use gaps in space, time, or frequency to transmit data, whereas in spectrum underlay networks, both PUs and SUs transmit data, provided that the PUs constrain the level of transmission power of the SUs in order to limit the level of interference. Meanwhile, the overlay method allows cooperative transmission of SUs and PUs, however, it is more complex to implement. Due to its simplicity and efficiency in improving spectrum utilization, using CRs in an underlay manner has been the most popular approach.

Power control algorithms play a vital role in spectrum underlay CR networks and can be implemented in either a distributed or centralized manner. The distributed approach is more practical due to two main reasons. First, it eliminates the need for a centralized controller in the network thus overcoming several drawbacks of centralized design, such as unreliability, excessive communication overheads, and limited scalability. Second, most devices which access the network are built by different manufacturers. As a result, it is hard to enforce those users to use pre-determined algorithms with hard-coded implementations [5].

SUs will generally have conflicting interests. This is due to the fact that each SU prefers to increase its transmit power as highly as possible in order to achieve higher data rates, and consequently obtain better quality-of-service (QoS). Unfortunately, this could increase the network interference thus motivating the need for the use of game-theoretic techniques to optimize power control in underlay CRNs [6]–[8].

Most of previous works on using game theory for CR [9]–[12] rely on the strong assumption on utility function of SUs, which means they use a global utility function that is not necessarily used by all the users in the network. For instance, in [9], authors propose a distributed scheme for power control in underlay cognitive radio networks, in which they define the summation of some concave functions as the objective function of the problem. However, there is no guarantee that autonomous, selfish users will in fact use such a utility function. In [10], a spectrum sharing scheme in which SUs share their private information to optimize their performance is proposed. The authors in [10] use cheat-proof strategies to punish users that deviate from cooperation. Although a cooperative model such as in [10] can achieve higher efficiency for spectrum sharing compared to fully noncooperative cases, it can also increase communication overhead. Thus, noncooperative models are more feasible and have attracted more interest for use in cognitive radio networks [11]. Pricing based methods have also received considerable attention for CR networks (CRNs) [12], [13]. Pricing is a mechanism used to enforce cooperation among SUs or between SUs and PUs. However, pricing mechanisms typically require significant overhead and communication resources. Clearly, the key limitations of existing power control mechanisms for CRNs include: reliance on unrealistic utility functions, the assumption of complete channel information for both PUs and SUs, and significant communication overhead in most of previous works.

The main contribution of this paper is a novel, fully distributed learning algorithm for optimizing power control in CRNs. The proposed algorithm does not require information exchange between the secondary users and relies on simple feedback from the PUs. In our proposed algorithm, each SU will dynamically update its strategy to achieve higher rates as well as conforming to PUs interference limitation. Then,
we cast the problem as a noncooperative game for which we derive the Nash equilibrium strategies and show that the proposed learning algorithm can find the unique equilibrium of the game. We also characterize the Nash equilibrium and show that the proposed algorithm converges to it. As a result, our key contributions include:

- We proposed update strategies based on bounded rationality using empirical estimation of competitors that have not been used in any related works so far. This decision-making approach forces users with more power to change their actions more often in case of interference.
- We show that the proposed learning algorithm can operate effectively without any channel state information or power strategy exchange between SUs. Only a simple feedback broadcast by PUs to all SUs within their range.
- The Nash equilibrium point of the game and the convergence conditions are obtained analytically.

The rest of this paper is organized as follows. The network model and the problem formulation is studied in Section II. Then, we present the algorithm and the game model for analyzing it in Section III. The analysis of the algorithm and its implementation are discussed in Section IV. In Section V, the analysis is validated through simulation results, and finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

The system model is illustrated in Fig. 1. Consider $M$ PU transceivers and $N$ SUs in a CR system. The channel gain from SU $k$ to PU $l$ is $g_{k,l}$ and the channel gain from SU $k$ to another SU $l$ is denoted by $h_{k,l}$. Channel state information (CSI) between each SU and PU is changing with time and is unknown to other SUs. Consider a synchronous time-slotted system. At each time slot $t$, each SU updates its transmitted power $p_k^t$ with regard to an interference limit which is constrained by PU base stations. Each PU base station is always active and has an interference limit $C_l$ such that:

$$\lim_{t \to \infty} \sum_{k=0}^{N} g_{k,l} p_k^t \leq C_l, \forall l = 1, 2, \ldots, M. \quad (1)$$

This means that the sum of interference experienced by each PU from the SUs should not violate the PU’s interference limit $C_l$. This interference is measured by each PU base station. The PU base stations are assumed to use orthogonal bands.

A. Problem Formulation

In the studied model, the SUs seek to maximize their rate while taking into account the PU’s interference limit $C_l$. This rate maximization is modeled as a constrained optimization problem for each SU. That is, each SU seeks to individually maximize its own capacity by solving the following optimization problem:

$$\max_{p_k} \quad W \log_2 \left(1 + \frac{K p_k h_{k,k}}{\sum_{i=1}^{N} p_i h_{i,k} + \sigma^2} \right),$$

s.t. $\sum_{k=1}^{N} g_{k,l} p_k \leq C_l; l = 1, \ldots, M, \quad (2)$

where $\sigma^2$ is noise power plus the interference caused by PUs on SUs. The transmit power of SU $k$ is denoted by $p_k$, and $W$ is the available bandwidth for the SUs. In addition, $K$ is the SINR gap between a practical data transmission rate and the Shannon capacity limit. Since the objective function in (2) is concave with respect to $p_k$ and the constraint is affine, the problem in (2) can be modeled as a convex optimization problem.

To solve this constrained optimization problem, one can write the Lagrangian and solve it in order to find the saddle point which represents the optimal solution for constrained optimization problems [14]. The standard Lagrangian for (2) is

$$f(p_k, \lambda_l) = -W \log_2 \left(1 + \frac{K p_k h_{k,k}}{\eta_k} \right) + \sum_{l=1}^{M} \lambda_l (g_{k,l} p_k + \eta_{k,l} - C_l); l = 1, \ldots, M, \quad (3)$$

in which the objective function for SUs is negated in order to change the maximization problem to a standard minimization problem and the terms $\eta_{k,l}$ and $\eta_k$ are defined as follows:

$$\eta_{k,l} = \sum_{i=1}^{N} p_i g_{i,l}, \quad \eta_k = \sum_{i=1}^{N} p_i h_{i,k} + \sigma^2. \quad (4)$$

For solving and finding the optimum saddle point $(p_k^*, \lambda_l^*)$, the following problem should be solved:

$$f(p_k^*, \lambda_l^*) = \min_{p_k} \max_{\lambda} \left\{ -W \log_2 \left(1 + \frac{K p_k h_{k,k}}{\eta_k} \right) \right\} + \sum_{l=1}^{M} \lambda_l (g_{k,l} p_k + \eta_{k,l} - C_l); \quad l = 1, \ldots, M, \quad (5)$$

In [15], a subgradient method for saddle-point problems has been proposed. However, this method is a centralized one. In this work, we model the optimization problem in (5) as a game which can be solved in a distributed manner. To this end, each
SU uses an iterative algorithm to optimize (5) with respect to its own power $p_k$ and each primary base station will optimize it with respect to its own Lagrange multiplier $\lambda_l$.

III. GAME THEORY FOR UNDERLAIRED CELLULAR CRN

A. Game Model

In order to solve the optimization problem stated in (5), in a distributed manner, we formulate the problem as a game

$$G = \{\mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}} \}$$

where:

1) $\mathcal{N}$ is the set of all users (SUs and PUs).
2) $\mathcal{S}_i$ is the continuous strategy set $[0, \infty)$ for the player $i$.
3) $(u_i)_{i \in \mathcal{N}}$ is the utility for the player $i$. For each SU player it is $-f(p_i, \lambda_l)$, and for each PU one is $f(p_i, \lambda_l)$ in (3).

B. Learning Dynamics

Here, we use a discrete algorithm for decision making which is the algorithm proposed in [16] for the so-called Cournot game, and is based on real economic systems production decisions. In the presence of a positive profit signal, bigger firms are more capable of making greater investments to increase their production, and in presence of negative profit signal, bigger firms are also more capable of decreasing their production in order to prevent damage.

To iteratively find the optimal solution of the convex problem in (5), we need each player to update its strategy according to a gradient-based algorithm. Therefore, we define the Lagrangian as a cost function for SUs and as a revenue function for PUs. Secondary users try to change their power in order to minimize their cost function while PUs will change parameter $\lambda_l$ in order to maximize their utility function. The change in $\lambda_l$ shows that how much PU is close to its interference limit and the closer the interference is to the limit of $l$th PU, the higher $\lambda_l$ will be. Thus, we have

$$p_{k}^{t+1} = p_k^t \left(1 - \gamma_k \frac{\partial f(p_k, \lambda_l)}{\partial p_k} \right),$$

$$\lambda_l^{t+1} = \lambda_l^t (1 + \beta_l \frac{\partial f(p_k, \lambda_l)}{\partial \lambda_l}).$$

With a proper choice of the learning rates $\beta_l$ and $\gamma_k$, neither $p_k$ nor $\lambda_l$ will become negative in the iterative process of the algorithm. Therefore, it can be shown that

$$p_{k}^{t+1} = p_k^t \left(1 + \gamma_k \frac{W K h_k k}{(\eta_k + K p_k h_k l) \ln(2)} - \sum_{l=1}^{M} \lambda_l^t (g_k) \right),$$

$$\lambda_l^{t+1} = \lambda_l^t (1 + \beta_l (p_k g_k l + \eta_k l - C_l)).$$

The proposed dynamics in (8) and (9) are update algorithms for SUs and PUs, respectively. However, since (8) needs other SUs’ strategy and channel state information we will use an empirical estimation of (6) instead of (8). This algorithm, for each SU and PU, is based on the local estimation of utility function $u_i$ that we defined earlier in the Subsection III-A. This local information is much easier to obtain compared to a global knowledge of the other users’ power or channel state information. We assume that SUs do not have complete information about the interfere that arises from other SUs and try obtain this kind of information by using empirical estimation. This estimation is obtained by using brief experiments of small power variations performed at the beginning of period $t$ and observing the change in their cost function, that is, their own rate and feedback value. This empirical estimation has been used in economic game models (see [17]). On the other hand, PUs can measure interference and then feedback the term $\lambda_l (I - C_l)$ where $I = h_{k,l} p_k + \eta_{k,l}$.

Given their local information, the secondary users behave as local cost minimizers. The local adjustment process is the one where a user decreases its power if it perceives a positive cost function variation ($\frac{\partial f(p_k)}{\partial p_k} > 0$), and increases its power if it perceived $\frac{\partial f(p_k)}{\partial p_k}$ is negative. Primary users also change their feedback value $\lambda_l$ according to rate of changes in $\frac{\partial f(p_k, \lambda_l)}{\partial \lambda_l}$. If the interference level PU is above the threshold, then $\frac{\partial f(p_k, \lambda_l)}{\partial \lambda_l} > 0$, and they will increase their feedback value $\lambda_l$ and if it is below that, then $\frac{\partial f(p_k, \lambda_l)}{\partial \lambda_l} < 0$, and they will decrease their feedback value. This adjustment mechanism has been called myopic.

Learning rate $\beta_l$ for $l$th PU’s updating algorithm has the concept that how much PUs are strict enforcing the interference limit, i.e. PU can adjust this factor to affect rate of PU’s conforming to interference limit. Also, for SU $k$, $\gamma_k$ is learning rate. Increasing both $\beta_l$ and $\gamma_k$ can increase convergence rate but it will increase instability of algorithm. In Section V, we will choose a diminishing learning rate for our algorithm. We summarized our learning algorithm for power control in Algorithm 1.

IV. NASH EQUILIBRIUM, QUASI-STATIONARITY AND CONVERGENCE

In this section, we discuss fairness, optimality and some implementation issues.

A. Pricing and Interference Control

The utility function in (5) which SU is trying to optimize is composed of two term: the first term is their rate function with respect to power and is a revenue function. The second term is a cost function that captures the fact that SUs will be charged a higher price for using more power. In other words, $F$ is the price of the power imposed by primary users and, as the interference increases, the price will increase. This price may be a part of the charge an SU pays or simply a control signal to guide PUs’ decisions.

B. Nash Equilibrium Existence, Uniqueness, and Characterization

First, we note that, for the proposed game, it can be shown that there exists a unique Nash equilibrium for the game, since the payoff functions for secondary users are concave with respect to their actions, there is a unique Nash equilibrium due to the strict diagonal concavity property [18].

The Nash equilibrium, by definition, is a stable state of a noncooperative game at which no player can earn more...
The Nash equilibrium for the game $G$ is given by the solution of:

\[
\begin{bmatrix}
    p_1 \\
p_2 \\
\vdots \\
p_N
\end{bmatrix}
= \begin{bmatrix}
p_1' \\
p_2' \\
\vdots \\
p_N'
\end{bmatrix} \eta = \tilde{p} \eta,
\]

where $\eta$ is normalization factor which is \(\sum_{k=1}^{N} C_k / \sum_{k=1}^{N} p_k g_{k,1}\) and

\[
\tilde{p} = \begin{bmatrix}
    K & h_{12} & \cdots & h_{1N} \\
    h_{12} & K & \cdots & h_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{1N} & h_{2N} & \cdots & K
\end{bmatrix}^{-1} \begin{bmatrix}
    k_1 \\
k_2 \\
\vdots \\
k_N
\end{bmatrix} - \frac{\sigma^2}{g_{NN} \lambda_1} \begin{bmatrix}
    h_{12} \\
    h_{22} \\
    \vdots \\
    h_{NN}
\end{bmatrix}.
\]

Proof: Using equilibrium state condition for SUs’ power, $p_k^{t+1} = p_k$ the following equations will be obtained:

\[
p_k \left( \frac{-k_1}{K p_k + \frac{1}{\lambda_k} \sum_{i \neq k} p_i h_{ik} + g_k \lambda_i} \right) = 0,
\]

for $k = 1, \ldots, N$, and using this condition for $\lambda$ parameter,

\[
\lambda_1^{t+1} \left( \sum_{i=1}^{N} p_i g_{il} - C_l \right) = 0 \quad k = 1, \ldots, N,
\]

where $k_1 = \frac{W K}{\ln 2}$. We derive the following equations by ignoring boundary solutions:

\[
k p_k + \frac{1}{h_{kk}} \sum_{i=1}^{N} p_i h_{ik} = \frac{k_1}{g_k \lambda_1} - \frac{\sigma^2}{h_{kk}},
\]

\[
\sum_{i=1}^{N} p_i g_{il} = C_l.
\]

Although equations are still nonlinear, by writing set of linear equations (14) in terms of $\lambda_1$, we derive

\[
\begin{bmatrix}
p_1 \\
p_2 \\
\vdots \\
p_N
\end{bmatrix} = \begin{bmatrix}
    K & h_{12} & \cdots & h_{1N} \\
    h_{12} & K & \cdots & h_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{1N} & h_{2N} & \cdots & K
\end{bmatrix}^{-1} \begin{bmatrix}
k_1 \\
k_2 \\
\vdots \\
k_N
\end{bmatrix} - \frac{\sigma^2}{g_{NN} \lambda_1} \begin{bmatrix}
h_{12} \\
h_{22} \\
\vdots \\
h_{NN}
\end{bmatrix}.
\]

Then, using (15), we can find $\lambda_1$ and then (10) will be obtained.

C. Time-Varying Environment

If the changes in interference limit for PUs and channel states is slow relative to the convergence of our learning algorithm, the proposed algorithm will still converge in a time-varying environment. Since we assumed slotted time, users update their strategy after specific time intervals. As a result, by decreasing this time interval, we are able to adapt with changes in channel states and interference limits of PUs. Doing so requires high-speed processing units in both secondary and primary users. It will not be a major problem in PUs’ base stations but SUs usually have limited processing power. As a result, our proposed algorithm can adapt to time varying environments with a maximum variation rate limited by SUs’ processing speed.

D. Convergence Analysis

Analyzing the convergence of the proposed learning algorithm is equivalent to stability analysis in control theory. When a fixed point of a dynamical system is stable, the system will converge to it. As discussed above, the Nash equilibrium is a fixed point of our nonlinear dynamics. Thus, if we show the local stability of this dynamical system, we can guarantee its convergence to the Nash equilibrium in a neighborhood of this point. In order to do so, we need to linearize dynamic
equations (8) and (9). Then we can check stability using eigenvalue analysis.

Suppose that we define state variables as follows:

\[
\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ p_1^1 \\ p_2^1 \\ \vdots \\ p_N^1 \end{bmatrix}.
\] (16)

Then, the update equations are given by:

\[
\mathbf{x}(t+1) = \mathbf{x}(t) + \mathbf{A}(\mathbf{x}(t))\mathbf{x}(t) + \beta x_1(t)\mathbf{B}\mathbf{x}(t),
\] (17)

where:

\[
\mathbf{A} = \text{diag}(-\beta C_1, \frac{c_1 \gamma_1}{\sum_{i=2}^{N+1} x_i(t) m_{i,1}}), \ldots, \frac{c_1 \gamma_N}{\sum_{i=2}^{N+1} x_i(t) m_{i,N}}),
\] (18)

\[
\mathbf{B} = \begin{bmatrix} g_{1,1} & g_{2,1} & \cdots & g_{N,1} \\ -\gamma_1 g_{1,1} & 0 & \cdots & 0 \\ \vdots & 0 & -\gamma_2 g_{2,1} & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & -\gamma_N g_{N,1} \end{bmatrix},
\] (19)

where \( m_{i,k} \) is defined as:

\[
m_{i,k} = \begin{cases} h_{i,k}, & \text{if } i \neq k, \\ K_i, & \text{if } i = k, \end{cases}
\]

and \( c_1 = \frac{W K}{\ln 2} \) is constant.

By linearizing the equation around the Nash equilibrium point, we have:

\[
\mathbf{x}(t+1) = \mathbf{Lx}(t),
\] (20)

where

\[
\mathbf{L} = \mathbf{I} + \beta x_1^*\mathbf{B} + \beta \mathbf{Bx}^* + \mathbf{A}(x^*) - c_1 \mathbf{D}x^* \mathbf{W},
\] (21)

and

\[
\mathbf{X}^* = \text{diag}(x_1^*, x_2^*, \ldots, x_N^*),
\] (22)

In addition, \( \mathbf{X}^* \) and \( \mathbf{W} \) in (21) are defined as follows:

\[
\mathbf{X}^* = \begin{bmatrix} x_1^* & 0 & \cdots & 0 \\ x_2^* & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x_N^* & 0 & \cdots & 0 \end{bmatrix},
\] (23)

\[
\mathbf{W} = \begin{bmatrix} 0 & \frac{\gamma_1 m_{11}}{(\sum_{i=2}^{N+1} x_i^* m_{i,1})^2} & \cdots & 0 \\ 0 & \frac{\gamma_1 m_{1N}}{(\sum_{i=2}^{N+1} x_i^* m_{i,N})^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \frac{\gamma_N m_{11}}{(\sum_{i=2}^{N+1} x_i^* m_{i,1})^2} & \cdots & \frac{\gamma_N m_{1N}}{(\sum_{i=2}^{N+1} x_i^* m_{i,N})^2} \end{bmatrix},
\] (24)

and \( \mathbf{I} \) is the identity matrix.

For the system to be stable, the eigenvalues of \( \mathbf{L} \) must lie inside the unit circle. Suppose \( \lambda(\mathbf{L}) \) is the set of eigenvalues of \( \mathbf{L} \), \( \lambda(\mathbf{L}) = \{\lambda_1, \cdots, \lambda_{N+1}\} \). By definition of matrix norm we know

\[
\max \left( \min_{i} \|\lambda_i\| \right) = \max \left( \min_{i} \|\lambda_i\| \right) \leq \|\mathbf{L}\| = \sup_{\|\mathbf{x}\|=1} \|\mathbf{Lx}\| \leq \|\mathbf{L}\| < 1.
\] (26)

Using the result above, we guarantee local asymptotic stability of the dynamics which guarantees the convergence to the Nash equilibrium.

### V. Simulation Results

We consider a case of four SUs and one PU base station that share a 1 MHz bandwidth. We use a Rayleigh fading channel model, therefore we consider \( h_{i,j}, i = j \) as Rayleigh random variables with unit variance, and for \( h_{i,j}, i \neq j \) and \( \sigma_i, i \) the variance is 0.1 (interference non-dominance condition). The interference limit for the PU is -110 dBm. We set \( K = 0.9 \). The model we consider for noise plus PU’s interference, is a white gaussian noise model with variance to 0.1. We also consider a diminishing learning rate.

Fig. 2 shows the convergence of the power for each SU over time. All the SUs are under the interference limit of the PU. We consider that the PU base station is always active. In (10), we obtained a closed form expression for the Nash equilibrium point whose theoretical outcomes are shown in Fig. 2. This is calculated for the PU’s interference limit and specific random CSI, with both of which the simulation runs. The rate convergence for each PU is also shown in Fig. 3. As we can see from both Fig.2 and Fig. 3, the algorithm will converge to Nash equilibrium in less than 200 steps.

Fig. 4 shows the convergence of our algorithm in the case of a sudden change of CSI at learning step 200. As discussed
in Section IV-C, the algorithm is able to overcome these changes in finite number of time steps. As we can see from Fig. 4 this finite number is 170 time-steps in our case. From Fig. 4, we can see that, for a given change in the channel state information of the system, the analytically derived Nash equilibrium will change accordingly. That is because Nash equilibrium in (10) depends on the CSI of PUs and SUs.

VI. CONCLUSION

In this paper, we have proposed a fully distributed learning algorithm for power control in underlaid CRNs which is designed for uncoordinated selfish SUs. We have particularly shown that the proposed algorithm can operate without any CSI or power strategy exchange among the SUs. We have shown the uniqueness of the Nash equilibrium using strict diagonal concavity condition, and the characterized analytically by solving nonlinear equation. Subsequently, we have proved that the proposed learning algorithm converges to the unique equilibrium. The asymptotic convergence to the Nash equilibrium point for the general case of $K$ users has also been proved using eigenvalue analysis. Simulation results have then corroborated our analytical derivations.

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