Game-Theoretic Approaches for Energy Cooperation in Energy Harvesting Small Cell Networks

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Abstract—Deploying small cells in cellular networks, as a technique for capacity and coverage enhancement, is an indispensable characteristic of future cellular networks. In this paper, two novel online approaches for enabling energy trading in multi-tier cellular networks with non-cooperative energy harvesting base stations (BSs) are proposed. The goal is to minimize the non-renewable energy consumption in a multi-tier cellular network with an arbitrary number of tiers. In the first approach, a decentralized energy trading framework is established in which BSs are stimulated to compensate their energy shortage by using the extra harvested energy from other BSs rather than using the non-renewable energy. Matching theory is used to assign BSs with energy deficit to the BSs with extra harvested energy. In the second approach, which is centralized, BSs with extra harvested energy and BSs with energy deficit enter a double auction for energy trading. The centralized approach also motivates the BSs with deficient energy to use other BSs extra harvested energy and satisfies a number of properties including truthfulness, individual rationalities and budget balance. Both approaches achieve Nash equilibrium and motivate non-cooperative BSs to share their extra harvested energy. The extra harvested energy is exchanged by the smart grid. We show that the amount of information exchanged in the network to enable BSs to trade energy is reduced in the centralized algorithm compared to the decentralized algorithm at the expense of using a control center. Simulation results verify that the proposed approaches reduce the non-renewable energy consumption conspicuously. Furthermore, by applying the proposed approaches, BSs gain more profit, and consequently, their utility functions enhance.

Index Terms—Multi-tier cellular network, energy harvesting, non-cooperative base stations, matching theory, double auction, energy trading, non-renewable energy.

I. INTRODUCTION

During the past decades, the amount of exchangeable information has been increased dramatically. The exponential growth in the wireless data traffic has motivated the need for new approaches to boost the capacity and coverage of wireless networks [3]. To solve this problem, small cells are employed to improve the overall wireless quality-of-service (QoS). This new type of cellular networks differs from traditional macro-cellular networks in many ways. Small cells have lower power and cover smaller areas. Depending on their capabilities, small cell BSs are classified into different types such as microcells, picocells and femtocells.

Deploying small cells raises many technical challenges in terms of resource allocation, outage management, distributed optimization and implementation. One of the fundamental challenges in small cell networks is energy management. In contrast to traditional cellular networks, base stations (BSs) of small cell networks differ in transmission power. Therefore, the amount of interference and the number of users served by each BS are different. As a result, the power management in this kind of cellular networks is different from traditional networks. Power control of small cells has been studied in the literature, e.g., [4]–[6]. In addition, green cellular networks have attracted lots of attention recently which is a way to reduce using up of fossil energy resources and it leads to reduction of greenhouse gases [7]. As a green technology, small cells with energy harvesting capabilities are alluring recent attention. In [8], a heterogeneous network is studied where BSs in different tiers are self-powered. If a BS has not harvested sufficient energy, it is kept OFF for charging, and its connected users are served by neighboring active BSs.

Energy cooperation in green networks is studied in [9], [10], where harvesting modules share a portion of their harvested energy with other harvesting modules. In [9] and [10], it is considered that the harvested energy is deterministic. Non-cooperative BSs are studied in the existing works in the literature. For instance, it is considered in [11] that there is no cooperation among BSs. There is no sharing of information among cells in [11]. This is the case that occurs when BSs belongs to different parties which pursue their own interests. Thus, if BSs are not recompensed, they have no incentive to cooperative. The assumption that the wireless network components operate non-cooperatively has also been studied in networks that route packets. In [12], an incentive scheme is proposed to motivate selfish nodes to cooperative to route packets from the source node to the destination in ad-hoc networks.

A. Related Work

With recent advancements in smart grid, two-way energy delivery between two nodes of the power network has been made possible [13]–[18]. The required smart infrastructures to enable two-way power and information delivery in a network are investigated in [13]. BSs in a cellular networks can be considered as energy loads in the smart grid [15], [16], [19]. In this paper, we use two-way energy delivery to design energy-efficient network to reduce the non-renewable energy consumption of BSs. The joint design and combination of the physical layer technique of coordinated multi-point (CoMP) with two way energy trading is studied in [19] where BSs are connected to the smart grid. In this paper, however, the harvested energy is not traded among BSs and the grid. Power transmission efficiency of transferring power from the remote power plant to an energy harvesting BS is much lower than transferring power among BSs [14]. Energy cooperation between cooperative BSs using
smart grid in multi-tier cellular networks is studied in our paper, 
[20], where the fairness of surplus renewable energy distribution 
is maximized. In [17], the authors study energy trading in a 
double-auction among storage units where energy demands are 
given prior to energy trades. In order to maximize its utility, 
each storage unit decides strategically on the amount of energy 
to sell in a local market. The energy exchange is done by 
the smart grid. The game is guaranteed to have at least one 
Nash equilibrium. We motivate non-cooperative BSs to trade 
energy among themselves by which one can combat the effect 
of the intermittent nature of the harvested energy from the non-
renewable energy source.

Market mechanism is a common approach to distribute re-
sources in wireless networks. Auctions and double auctions as 
mechanisms to design markets have been studied widely in order 
to distribute resources like spectrum, bandwidth and energy 
among agents [17], [21]–[23]. The authors in [17] propose a 
framework in which a double auction is combined with a non-
cooperative game. In contrast to our model, energy demands are 
given prior to energy trades in [17]. In our paper, the goal is 
minimizing the non-renewable energy consumption and energy 
trades are designed to meet this goal, however, in [17] the 
demand is not always compensated by the supply as much as 
possible. The reason is that bids of sellers are chosen randomly 
in [17]. The amount of traded energy and gained profits of 
storage units are stochastic consequently. As the amount of 
traded energy is stochastic, it fails to compensate energy deficit 
of BSs thoroughly in most of time slots. Moreover, the priority 
of selling excessive energy is not assigned to those storage units 
that their surplus energy is close to their battery capacity. In this 
case, the waste of energy increases due to the energy overflow 
in the limited battery of the energy storage units.

We propose algorithms that use battery level information, the 
correlation of the harvested energy by a BS and the correlation 
of the demanded energy from a BS in time slots. In none of the 
refereed papers, near BSs are assigned to each other in order to 
reduce the smart grid usage for transferring energy.

B. Contributions

In this paper, we study the reduction of the non-renewable 
energy consumption in multi-tier cellular networks with energy 
harvesting where the harvested and the demanded energy of BSs 
are stochastic. To reduce the non-renewable energy consump-
tion, non-cooperative BSs are motivated to share their extra 
harvested energy with BSs that have not harvested sufficient 
energy. Two online algorithms are proposed to motivate non-
cooperative BSs to share their extra harvested energy. In the 
first algorithm, which is decentralized, after BSs utility functions 
are defined, the prices of energy trades are found and BSs 
trade energy among themselves without requiring any control 
center. By using the matching theory, BSs with energy deficit are 
matched to BSs with the extra harvested energy. In the second 
algorithm, which is centralized, BSs with energy deficit and BSs 
with the extra harvested energy are entered to a double auction 
in order to trade energy. The double auctions introduced in [24] 
and [25] can be used to distribute energy. However, when the 
size of the network increases, the size of optimizations given 
in [24] and [25] increase dramatically. Therefore, they are not 
applicable for online applications. To reduce smart grid usage 
for transferring energy as well as reducing its usage cost, an 
optimization is embodied in the proposed algorithm to assigned 
BSs with energy deficit to near BSs with the extra harvested 
energy for trading energy. Both algorithms stimulate BSs with 
the extra harvested energy to trade energy. We compare the 
performance of our approaches with the current approaches 
existing in the literature which shows the efficiency of the 
approaches. We A summary of this paper contributions is: 1) we 
propose utility functions for BSs and establish an energy trading 
framework accordingly; 2) in the first algorithm, we propose 
appropriate transaction fees for energy trades and by using them, 
we control prices of energy trades. It is shown that the price of 
each energy trade is the Nash equilibrium. We demonstrate that 
the price of the harvested energy per Joule shared by BSs is 
cheaper than the price of one Joule of the non-renewable energy.

Therefore, BSs with energy deficit are motivated to buy the 
required energy from other BSs; 3) we reflect the fact that both 
the harvested energy by a BS and the demanded energy from a 
BS are correlated in time to energy trades prices. BSs with 
high extra harvested energy sell their energy at low prices. This 
cumulates in selling extra energy quickly which prevents from 
possible waste of energy due to the limited battery capacities;
4) in the second algorithm, which is centralized, to show differences of seller BSs with respect of the amount of the extra 
stored energy, and to show differences of buyer BSs with respect 
of the needed energy, each BS, seller or buyer, is provided with 
a variable called type; 5) based on defined BSs types, BSs have 
different valuations for their extra or required energy. Therefore, 
the priority in selling energy is assigned to BSs with high extra 
harvested energy which prevents the possible waste of energy 
due to the limited battery capacities. BSs are entered to a double 
auction for energy trading; 6) we find the best response of BSs in 
bidding. It is shown that the second approach satisfies a number 
of properties like truthfulness, individual rationalities and budget 
balance that keep BSs motivated to enter the double auction 
for energy trading. Moreover, we show that the double auction 
reaches the Nash equilibrium when BSs do their best responses;
7) it is shown that the amount of information needed to be 
distributed in the network to enable BSs to trade energy reduces 
in the centralized algorithm compared to the decentralized one 
at the expense of using a control center; 8) a closed-form for 
the lower bound of the expected obtained revenue from selling 
an energy portion is found; 9) in both algorithms, we design 
energy trading schemes such that BSs with energy deficit are 
assigned to near BSs with extra harvested energy which reduces 
the usage of the smart grid for distributing green energy; 10) in 
both of the proposed algorithms, BSs with high extra harvested 
energy sell their energy at low prices. This leads to selling extra 
energy quickly which prevents the possible waste of energy 
due to the limited battery capacities. Moreover, the priority 
for buying energy is assigned to BSs with high energy deficit 
which increase energy distribution fairness; 11) we find the 
communication overhead of both of the proposed approaches 
as well as their computational complexities.

The rest of this paper is organized as follows: Section II 
presents the model of the studied small cell network and 
harvested energy by BSs. In Section III, the decentralized energy 
trading scheme among BSs is given and it is demonstrated that 
the prices of energy trades are Nash equilibrium. The Section IV 
presents the centralized energy trading scheme using a double 
auction mechanism. We demonstrate a number of properties 
for the proposed energy trading and find BSs best response. 
Simulation results are given in Section V and we conclude this 
paper in Section VI.

II. SYSTEM MODEL

Consider the downlink of a multi-tier cellular wireless net-
work consisting of BSs, classified into K tiers. BSs belonging 
to tier k have maximum transmit power of $P_k$. Serving the
connected users causes the power depletion in BSs. We ignore other types of energy consumption at the BSs. We assume that the location distributions of the BSs operating in different tiers are approximated as independent Poisson Points Processes (PPPs) \[26\] with density \( \lambda_k, k \in \{1, \ldots, K\} \). The reason is that small cell BSs locations are generally unplanned and so are well-modeled by a spatial random process \[27\]–\[29\]. In addition, it is included in \[26\] that macrocell BSs may also be reasonably well modeled by a random spatial point process, with about the same as accuracy as the typical grid model. Each BS is equipped with an individual energy harvesting module and an energy storage device. We assume that the BSs belonging to given tier \( k \) have similar battery capacity \( c_k \). In the considered network, the distribution of the users’ location also follows a PPP with density \( \lambda_u \). Here, the users are allowed to connect to any tier and each user connects to the BS that provides the highest long term received power. In other words, small scale fading gain does not affect the cell selection. Depending on the densities of the tires and their transmit powers, the average number of users connected to a BS of tier \( k \) is given by \[8\] Corollary 1:

\[ N_k = \frac{\mu_k \lambda_k \|X^k\|^2}{\sum_{j=1}^{K} \lambda_j \|X^j\|^2} \quad \|X^k\|^2, \]  

(1)

where \( P_k \) is the coverage probability. The coverage probability denotes the portion of the users connected to a BS with SIR above a threshold. The path loss exponent is shown by \( \gamma \), which typically lies in the range of \( 2 \leq \gamma \leq 6 \).

### A. The Demanded Energy Model

The traffic rate demand of user \( m \) connected to the \( i \)th BS of tier \( k \) at time slot \( t \) is denoted by \( R_{m}^{i,k}(t) \) and it is constant during the time slot. Users may request different rates through the time slots. The total rate that BS \( i \) in tier \( k \) has to serve is \( R_{i,k}(t) = \sum_{m=1}^{N_{i,k}(t)} R_{m}^{i,k}(t) \), where \( N_{i,k}(t) \) denotes the number of connected users and it is a Poisson random variable \[31\]. Its mean value is given in \[1\]. The requested rates of users are considered to be stochastic. The consumed power of the \( i \)th BS of tier \( k \) at time slot \( t \) to provide user \( m \) with rate \( R_{m}^{i,k}(t) \) is denoted by \( p_{n}^{i,m}(t) \). Due to the stochastic nature of the demanded rates, the consumed powers of the BSs are stochastic. The consumed power to serve each user connected to a BS is modeled as an arbitrary correlated random process due to the correlation in user traffic and usage patterns \[10\]. Serving all connected users to a BS consumes \( p^{i,k}(t) = \sum_{m=1}^{N_{i,k}(t)} p_{n}^{i,m}(t) \) Watts. Hence, \( p^{i,k}(t) \) is a function of two kinds of random variables, \( p_{n}^{i,m}(t), \forall m, \) and \( N_{i,k}(t) \). The consumed energy at time slot \( t \) is obtained as

\[ \int_0^T p^{i,k}(t) \, dt = T p^{i,k}(t) = T \sum_{m=1}^{N_{i,k}(t)} p_{n}^{i,m}(t) \text{ where } T \text{ is the time slot duration and the demanded power is constant in each time slot. The energy stored in the battery of the BS } i \text{ in tier } k \text{ at time slot } t \text{ is denoted by } e^{i,k}(t). \text{ The shortage of energy in the BS battery necessitates buying energy from other BSs or using the energy of the non-renewable source.}

### B. The Harvested Energy Model

The harvested energy by each BS can be modeled as an arbitrary correlated random process. Let \( \mu_{i,k}(t) \) denote the total amount of the harvested energy at the \( i \)th BS of tier \( k \) during time slot \( t \). It is considered in \[8\] that the BSs across tiers may differ both in terms of how fast they harvest energy, and how much energy they can store which is \( c_k \) joules. If the harvested energy by a BS is more than its needs, the extra energy is stored at its battery. The energy is wasted when the BS tries to store more energy than the battery capacity. Therefore, the BS that has harvested energy more than its needs is motivated to trade the extra energy. In our model, the BSs are allowed to sell the extra harvested energy to the BSs that have energy shortage. If a BS, which is in need of energy, finds no seller BS, it has to use the non-renewable energy to serve the connected users. It is also assumed that BSs are connected to the smart grid. The smart grid is a technology that enables more precise measurement of the electric power by using smart devices which can communicate with each other. When the BSs trade their energy by using the smart grid, the smart grid operator charges a cost for such service. This cost is an increasing function of the distances as well as the amount of the traded energy \[32\]. In our model, we assume that distances among BSs are known. When two BSs trade energy, energy is transferred by the smart grid and no energy is consumed by BSs while they trade energy. Each BS knows distances among itself and other BSs. The stored energy is updated as

\[ e^{i,k}(t + 1) = \min\{\max\{e^{i,k}(t) - T p^{i,k}(t), 0\} + \mu_{i,k}(t) \pm E_T(t + 1), c_k\}, \]  

where \( E_T(t + 1) \) is the amount of the traded (shared) energy at the beginning of time slot \( t + 1 \) and it is added if the BS is buyer, or it is subtracted if the BS is seller. In \[2\], the maximum stored energy in the battery is equal to the battery capacity. Furthermore, the BS cannot use more energy than the stored amount from its battery. To enable a BS to compensate its energy deficit by the harvested energy of other BSs, a framework for the energy trading is established in the following sections.

### III. ENERGY TRADING SCHEME AMONG BSs USING A MATCHING ALGORITHM

Energy harvesting is not a reliable source of energy for the BSs due to the uncertainty in the environmental conditions. In order to minimize the non-renewable energy consumption, we stimulate BSs to share their extra harvested energy with other BSs which are in shortage. At the beginning of each time slot, each BS broadcasts a message to other BSs. This message contains the information of the tier that the BS belongs to, its battery level and the amount of extra or needed energy to serve its connected users in that time slot. Based on these information, all BSs are classified into two categories, i.e., the seller BSs and the buyer BSs. The seller BSs have extra stored energy and the buyer BSs have energy shortage. The set of the seller BSs of tier \( k \) is denoted by \( S_k = \{s_{k}^1, s_{k}^2, \ldots, s_{k}^{n_k}\} \) and the set of buyer BSs of tier \( k' \) is \( B_{k'} = \{b_{k'}^1, b_{k'}^2, \ldots, b_{k'}^{n_{k'}}\} \), where \( n_k \) and \( n_{k'} \) are the number of seller and buyer BSs of tiers \( k \) and \( k' \), respectively. The set of seller BSs is \( S = \bigcup_{k=1}^{K} S_k \). The set of buyer BSs is \( B = \bigcup_{k'=1}^{K} B_{k'} \). Since it is known that a BS is seller or buyer at the beginning of the time slot, one can display the battery level, the number of connected users and the demanded power from a BS by using the BS index in the set of sellers or buyers of the tier that the BS belongs to, \( S_k \) or \( B_{k'} \). Consequently, \( e^{k}(t) \) and \( N^{k}(t) \) are used for a seller BS \( s_{k}^i \), and \( p^{k}(t) \), \( N^{k'}(t) \) and \( p^{k'}(t) \) are used for buyer BSs \( b_{k'}^j \) of tier \( k' \), respectively. The amount of energy that a seller BS \( s_{k}^i \) from tier \( k \in \{1, 2, \ldots, K\} \) is willing to sell at time slot \( t \) is

\[ \rho^{k}(t) = e^{k}(t) - T p^{k}(t). \]  

(3)
A buyer BS \( b_{k'} \), from tier \( k' \in \{1, 2, \ldots, K\} \) wants to buy

\[
\rho_{k'}(t) = \min \left\{ \left[ (e^{k'}(t) - T p^{k'}(t)) \right], T P_{k'} \right\}.
\]

The maximum energy that a BS in tier \( k' \) can consume to serve users is \( T P_{k'} \). The total extra energy stored in BSs of the network at time slot \( t \) is \( \sum_{k=1}^{K} \sum_{j=1}^{n_k} \rho_{k}(t) \) and the total needed energy of BSs with energy deficit is \( \sum_{k=1}^{K} \sum_{j=1}^{n_k} \rho_{k}(t) \). Each BS serves a number of users connected to it, and the consumed energy to serve them costs a known price of \( \zeta \) units of money per Joule. A BS receives requests from the users at the beginning of each time slot. Deployed BSs in the network are considered to behave in a non-cooperative manner. In other words, regardless of other BSs, a BS wants to earn money by serving the connected users. Non-cooperative BSs do not share energy to help BSs with energy deficit since their energy may become useful for serving users in future time slots. Earning money is an incentive for a seller BS to share its extra energy. The smart grid usage to trade energy is not free for BSs. The cost of transferring energy by the smart grid depends on the distance between the seller and the buyer BS, denoted by \( g_{k',b_k}(\delta) \), per meter per Joule. It is assumed that the cost of energy transfer is a linear function of the amount of the transferred energy, \( E_T(t) \), and an arbitrary increasing cost function of distance denoted by \( \Gamma(g_{k',b_k}(\delta)) \). The cost of sharing \( E_T(t) \) Joules is captured by \( E_T(t) \Gamma(g_{k',b_k}(\delta)) \) and it is paid by the buyer BS. Consider a seller BS \( s_k \) operating in the \( k \)-th tier consuming \( p^{k}(t) \) Watts to serve its users. The BS \( s_k \) earns \( \zeta T p^{k}(t) \) revenue from consuming \( T p^{k}(t) \) Joule. Its utility function at time slot \( t \) is defined as follows

\[
U^{\rho_{k}}(\eta(E_T(t)), t) = \zeta T p^{k}(t) + \eta(E_T(t)),
\]

where \( \eta(E_T(t)) \) is the price of \( E_T(t) \) units of shared (sold) energy. The surplus energy of a BS is injected to the grid by using smart infrastructure. In the next subsection, the appropriate price of \( \eta(E_T(t)) \) is obtained by using a game-theoretic approach. The seller BS has enough energy to serve users connected to it. Consequently, it serves \( N_{s_k}(t) \) users and it sells its extra energy to gain utility of size \( \eta(E_T(t)) \).

Similarly, we define the utility function of the buyers. The buyer BS gains utility by serving its connected users which consumes \( T p^{k'}(t) \) Joule. The energy of the buyer BS is not enough to serve all the connected users. The utility function of a buyer BS at time slot \( t \) is obtained as

\[
U^{\rho_{k'}}(\eta(E_T(t)), t) = \zeta T p^{k'}(t) - \eta(E_T(t)) - E_N(t) \psi - E_T(t) \Gamma(g_{k',b_k}(\delta)),
\]

where \( E_N(t) = \rho_{k'}(t) - E_T(t) \) is the amount of energy which is not bought from \( s_k \) and is purchased from the non-renewable source. The price that the buyer BS pays to the non-renewable source is \( E_N(t) \psi \) where \( \psi \) is the price of the non-renewable energy per Joule. To motivate BSs to consume renewable energy rather than the non-renewable energy, the price of the non-renewable energy has to be greater than the price of the renewable energy. Otherwise, the BSs use the non-renewable energy to compensate their needs. We assume that \( \psi \) should satisfy the condition \( \zeta + \max_{g_{k',b_k}(\delta)} \Gamma(g_{k',b_k}(\delta)) < \psi \), \( \forall s_k \in S \), \( \forall b_{k'} \in B \).

In the case that \( \zeta, \psi \) and the smart grid usage cost are such that \( \zeta + \max_{g_{k',b_k}(\delta)} \Gamma(g_{k',b_k}(\delta)) < \psi \), \( \forall s_k \in S \), \( \forall b_{k'} \in B \) is not satisfied, the network is divided into the minimal number of smaller areas such that in each of the yielded areas the above inequality is satisfied. In that case, \( \max_{g_{k',b_k}(\delta)} \Gamma(g_{k',b_k}(\delta)) \) is smaller compared to the larger network. In the above inequality, the supremum of the price of one Joule bought from other BSs plus the cost of transferring it is lower than the price of one Joule of the non-renewable energy. In this case, the matching-game-based algorithm and the double-auction-based algorithm can perform to carry out energy trades in the smaller areas. When the network is divided into minimal number of areas, the non-renewable energy consumption increases compared to the case that it is not. The reason is that each BS loses the chance to trade energy with BSs located in the other subnetwork. When a BS receives energy from grid, it uses smart infrastructure. Similar to [13]–[18], and [19], we assume that the loss of energy trading is negligible asymptotically.

The energy sharing among BSs reduces the non-renewable energy consumption of BSs. In order to enable BSs to exchange money, a Credit Clearing Service (CCS) is used [12], [33], where all BSs have credit accounts with initial funds. After a seller and a buyer BSs agree on a price, the price, tiers of both BSs, their battery levels, the needed energy of the buyer and the

| \( P_k \) | maximum transmit power of tier \( k \) |
| \( \lambda_u \) | density of users |
| \( K \) | number of tiers |
| \( n_k \) | number of buyer BSs in tier \( k' \) |
| \( N_{s_k}(t) \) | number of connected users to \( s_k \) BS in tier \( k \) at time slot \( t \) |
| \( p^{k}(t) \) | power consumption of \( s_k \) BS in tier \( k \) at time slot \( t \) |
| \( c_k \) | battery capacity of tier \( k \) |
| \( B \) | set of buyer BSs |
| \( b_j \) | needed energy of \( j \)-th buyer BS in tier \( k' \) |
| \( \eta \) | price of traded energy |
| \( \delta \) | distance between \( s_k \) and \( b_{k'} \) |
| \( u_{b_{k'}}(t) \) | number of portions that one Joule is divided to |
| \( \chi_{b_{k'}}(t) \) | type of BS \( b_{k'} \) at time slot \( t \) |
| \( v_{k}^{q} \) | value of the \( q \)-th extra energy portion for \( s_k \) |
| \( z_{q}^{k} \) | revenue \( s_k \) receives for selling the \( q \)-th extra energy portion |

| \( \gamma \) | path loss exponent |
| \( \lambda_k \) | density of BSs in tier \( k \) |
| \( n_k \) | number of seller BSs in tier \( k \) |
| \( e^{k}(t) \) | stored energy in the battery of \( k \)-th BS in tier \( k \) at time slot \( t \) |
| \( E_T(t) \) | amount of traded energy |
| \( \mu^{k}(t) \) | harvested energy of \( k \)-th BS in tier \( k \) at time slot \( t \) |
| \( S \) | set of seller BSs |
| \( \rho_k \) | surplus energy of \( k \)-th seller BS in tier \( k \) |
| \( \psi \) | price of consuming one Joule at a BS for a user |
| \( \Gamma \) | price of using smart grid |
| \( \zeta \) | type of BS \( s_k \) at time slot \( t \) |
| \( \chi^k(t) \) | number of sold extra energy portions of \( s_k \) at time slot \( t \) |
| \( w_q^{k} \) | revenue \( s_k \) receives for selling its \( q \)-th extra energy portion |
| \( \chi^{b_{k'}}(t) \) | value of the \( q \)-th needed energy portion for \( b_{k'} \) |
| \( z_q^{b_{k'}} \) | revenue \( b_{k'} \) pays for buying its \( q \)-th needed energy portion |
extra energy of the seller are submitted to the CCS as the trade characteristics. The CCS moves the money from the buyer BS credit account to the seller BS credit account according to the agreed price. The CCS controls the trade prices as well. It is assumed that the distances among BSs are known by the CCS. Assume that the buyer BS $b_{k'}$ starts negotiating with the seller BS $s_k$ as rational game players. Since the buyer BS wants to compensate all of its energy deficit, the amount of the shared energy is the minimum of the extra stored energy in the seller BS and the needed energy of the buyer BS, $\min\{\rho^{s_k}(t), \eta^{b_{k'}}(t)\}$.

The BS $s_k$ proposes a price to the BS $b_{k'}$ to maximize its utility function. The utility function of the BS $s_k$ is an increasing linear function of $\eta(E_{T}(t))$. Thus, by proposing a higher price, its utility increases. The BS $s_k$ can obtain the utility function of the BS $b_{k'}$, since the BS $b_{k'}$ broadcasts its tier, its battery level and its amount of needed energy at the beginning of time slots. Moreover, the cost of transferring energy by the smart grid and the amount of shared energy are known. If the price, $\eta(E_{T}(t))$, is greater than or equal to $E_T(t)$, the BS $s_k$ does not accept it. The reason is that this price results in a lower or equal utility than the utility yielded from buying energy from the non-renewable source, i.e.,

$$\zeta T p^{b_{k'}}(t) - \eta(E_{T}(t)) - E_T(t) \psi - E_T(t) < E_T(t) \Gamma(g^{s_k, b_{k'}})$$

To restrict the proposed price by a seller BS and keep buyer BSs motivated to exploit the renewable energy, we employ the transaction fee concept introduced in [12]. This is a fee paid by the BS $s_k$ to the CCS for updating accounts after the agreement. The utility function of the seller BS is revised as

$$U^{s_k}(\eta(E_{T}(t)), t) = \zeta T p^{s_k}(t) + \eta(E_{T}(t)) - F(\eta(E_{T}(t))),$$

where $F(\eta(E_{T}(t)))$ is the transaction fee. According to the seller and buyer battery levels, the needed energy of the buyer and the extra energy of the seller, the CCS can find the utility functions of seller and buyer BSs and the price of the shared energy. Using the transaction fee, the CCS is able to affect the proposed price of the seller BS to keep the buyer BS motivated to use the shared energy and avoid wasting the harvested energy. When the extra stored energy in the battery of the seller BS approaches the battery capacity, the proportion of the extra stored energy of the seller BS to its battery capacity, $\rho^{s_k}(t)$, is near one. Based on the battery level, two cases are considered. In the first case, $\rho^{s_k}(t)$ is less than $\vartheta$ where $\vartheta$ is an arbitrary parameter such that $0 \leq \vartheta \leq 1$. In the other case, $\rho^{s_k}(t) \geq \vartheta$. In this case, the CCS forces the seller BS to offer a lower price. It culminates in selling the extra energy quickly which reduces possible waste of energy in the next time slots due to the correlation between the harvested energy. As the battery capacity of the seller BS affects the price of the shared energy, BSs are classified into tires.

1) The proportion of the extra stored energy of a seller BS to its battery capacity is less than $\vartheta$: In this case, the stored energy in the seller BS $s_k$ is not close to the battery capacity. The utility that the buyer BS $b_{k'}$ gains by consuming one Joule at time slot $t$ is $\zeta T p^{b_{k'}}(t) - \eta(E_{T}(t)) - E_T(t) = E_T(t) \Gamma(g^{s_k, b_{k'}})$ units of money. Therefore, it gains $E_T(t) \zeta T p^{b_{k'}}(t) - \eta(E_{T}(t)) - E_T(t) = E_T(t) \Gamma(g^{s_k, b_{k'}})$ by consuming $E_T(t)$ units of energy. Consider that $\beta$ is an arbitrary parameter such that $0 \leq \beta \leq 1$. The CCS asks a small transaction fee $\vartheta$ where $\vartheta \to 0$ if the BS $s_k$ proposes a price to gain utility up to $\beta \times 100$ percent of the utility that the BS $b_{k'}$ gains by consuming $E_T(t)$ units of energy. If the seller proposes a higher price, the CCS asks the high transaction fee $\frac{1}{\beta}$. Therefore, $F(\eta(E_{T}(t)))$ is defined as follows

$$F(\eta(E_{T}(t))) = \begin{cases} \theta, & \text{if } \eta(E_{T}(t)) \leq \beta E_T(t) \\ \frac{\zeta T p^{b_{k'}}(t) - \eta(E_{T}(t)) - E_T(t)}{E_T(t) \Gamma(g^{s_k, b_{k'}})} \times T p^{b_{k'}}(t), & \text{otherwise.} \end{cases}$$

(9)

The above transaction fee is rewritten by keeping the common term, $\eta(E_{T}(t))$, in the left side of the inequality as follows

$$F(\eta(E_{T}(t))) = \begin{cases} \vartheta, & \text{if } \eta(E_{T}(t)) < \beta E_T(t) \\ \frac{\zeta T p^{b_{k'}}(t) - \eta(E_{T}(t)) - E_T(t)}{E_T(t) \Gamma(g^{s_k, b_{k'}})} \times T p^{b_{k'}}(t) + 1, & \text{otherwise.} \end{cases}$$

(10)

When the BS $s_k$ increases its proposed price to $b_{k'}$, greater than $\eta(E_{T}(t))$ the smart grid usage price, $E_T(t) \Gamma(g^{s_k, b_{k'}})$, is lower than the price of buying $E_T(t)$ Joule from the non-renewable source. As $\psi E_T(t) + E_T(t) \Gamma(g^{s_k, b_{k'}})$ is positive, by adding it to the numerator of the above transaction fee, we have $F(\eta(E_{T}(t)))$ given in (11), the resulted fracture increases and the subsequent inequality in (11) holds. Next, when we remove the term 1 from the denominator in (11), the new resulted fracture given in (12) increases. We shall notice that $0 \leq \beta \leq 1$. Consequently, $E_T(t)|\beta + \Gamma(g^{s_k, b_{k'}})| \leq E_T(t)|\vartheta + \Gamma(g^{s_k, b_{k'}})|$ holds. Since the smart grid price is such that $\vartheta + \max\left\{\psi E_T(t) + E_T(t) \Gamma(g^{s_k, b_{k'}}), \psi E_T(t) + E_T(t) \Gamma(g^{s_k, b_{k'}})\right\}$, the buyer finds it cheaper than the non-renewable energy price, the proposed price is the Nash equilibrium.

Consider that in a time slot, the stored energy by the BS $b_{k'}$ is small such that $\zeta T p^{b_{k'}}(t) - E_N(t) \psi - E_T(t) < E_T(t) \Gamma(g^{s_k, b_{k'}})$ by
\[ \eta(E_T(t)) + E_T(t) \Gamma(g^{s_i,b_{j'}}) = \frac{\zeta T \rho^{b_{j'}}(t) - \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}})}{T \rho^{b_{j'}}(t) + 1} + E_T(t) \Gamma(g^{s_i,b_{j'}}) < \frac{\zeta T \rho^{b_{j'}}(t)}{T \rho^{b_{j'}}(t) + 1} + E_T(t) \Gamma(g^{s_i,b_{j'}}) \]  

(11)

\[ \leq \frac{\zeta T \rho^{b_{j'}}(t)}{T \rho^{b_{j'}}(t) + 1} + E_T(t) \Gamma(g^{s_i,b_{j'}}) = E_T(t) \left[ \zeta + \Gamma(g^{s_i,b_{j'}}) \right] \leq E_T(t) \left[ \zeta + \Gamma(g^{s_i,b_{j'}}) \right]. \]  

(12)

Then the seller has no incentive to share its energy with the BS \( b_{j'} \). Therefore, the CCS applies the following transaction fee

\[ F(\eta(E_T(t)), t) = \begin{cases} \theta, & \text{if } \eta(E_T(t)) \leq E_T(t) \zeta \\ 1, & \text{otherwise,} \end{cases} \]  

(13)

where \( \theta \rightarrow 0 \). When \( \zeta T \rho^{b_{j'}}(t) = 1 + \theta - \frac{\psi E_N(t)}{c_k} \) is applied as the transaction fee, the BS \( s_i \) increases its utility by proposing \( \eta(E_T(t)) = E_T(t) \zeta \) and the buyer BS accepts it. The seller avoids proposing more price than \( E_T(t) \zeta \) due to the high transaction fee \( \frac{1}{\theta} \). Although this price makes the buyer BS utility negative, it is better than buying energy from the non-renewable source at price \( E_T(t) \psi \). The reason is that \( E_T(t) \left[ \zeta + \max_{g^{s_i,b_{j'}}} \Gamma(g^{s_i,b_{j'}}) \right] < E_T(t) \psi \).

As always \( \min \left\{ \zeta T \rho^{b_{j'}}(t) \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}}) \right\} \leq E_T(t) \zeta \), the transaction fee \( \zeta T \rho^{b_{j'}}(t) \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}}) \) motivates the seller BSs to assign the first priority in sharing energy to a buyer BS that has harvested small energy which increases the fairness of the energy distribution. The BS \( b_{j'} \) can pay the price from its credit account. If the seller and buyer BSs change their actions, they cannot gain more utility and the proposed price, \( \eta(E_T(t)) = E_T(t) \zeta \), is the Nash equilibrium.

2) The proportion of the extra stored energy of a seller BS to its battery capacity is greater than or equal to \( \psi \). In this case, the extra stored energy of the seller BS is near the battery capacity and \( \theta < \frac{\psi E_N(t)}{c_k} \). The CCS scales the transaction fee thresholds in \( [10] \) and \( [13] \) by a positive multiplier less than one such as \( 1 + \theta - \frac{\psi E_N(t)}{c_k} \) to force the seller BS to propose a lower price compared to the previous case. The more \( \frac{\psi E_N(t)}{c_k} \) gets close to one, the thresholds are reduced more. This policy results in proposing lower price which helps to sell the extra energy quickly that prevents from the waste of energy in the following time slots due to the correlation in the harvested energy values. If \( \zeta T \rho^{b_{j'}}(t) - \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}}) > 0 \), the seller BS maximizes its utility by proposing

\[ \eta(E_T(t)) = \frac{1 + \theta - \frac{\psi E_N(t)}{c_k}}{1 + \theta - \frac{\psi E_N(t)}{c_k}} \times \frac{\zeta T \rho^{b_{j'}}(t) \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}})}{T \rho^{b_{j'}}(t) + 1}. \]  

(14)

The reason is that if the BS \( s_i \) proposes a price higher than \( \frac{\zeta T \rho^{b_{j'}}(t) \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}})}{T \rho^{b_{j'}}(t) + 1} \), its gained utility decreases greatly by the transaction fee \( \frac{1}{\theta} \). Since \( 1 + \theta - \frac{\psi E_N(t)}{c_k} < 1 \), it is straightforward to demonstrate this price is lower than the non-renewable energy price. The buyer BS accepts this price since it is cheaper than the non-renewable energy price. Therefore, if the involved BSs change their actions, they cannot gain higher utilities and the price is the Nash equilibrium. With the same reasons given in the previous case when \( [13] \) is applied as the transaction fee, if \( \zeta T \rho^{b_{j'}}(t) - \psi E_N(t) - E_T(t) \Gamma(g^{s_i,b_{j'}}) \leq 0 \), the proposed price and the Nash equilibrium is \( \eta(E_T(t)) = (1 + \theta - \frac{\psi E_N(t)}{c_k}) E_T(t) \zeta \). This price is obtained after the threshold in \( [13] \) is scaled with \( 1 + \theta - \frac{\psi E_N(t)}{c_k} \).

Comparing two prices obtained in both cases, it is observed that when the transaction fee threshold is scaled by \( 1 + \theta - \frac{\psi E_N(t)}{c_k} \), the proposed price decreases. The reason is that \( 1 + \theta - \frac{\psi E_N(t)}{c_k} < 1 \). In the second case as the battery level is close to battery capacity, the energy is priced cheaper. Since the buyer BSs are rational and they prefer to buy energy at a lower price, they request BSs offer cheap energy first and this results in selling extra stored energy quickly. Selling energy quickly prevents possible energy waste due to the limited battery.

A. Matching the Buyer and the Seller BSs

Based on the BSs tiers, battery levels and the extra or the needed energy, a buyer BS starts finding the price of the shared energy with rational seller BSs as explained in previous section. Next, the buyer BS ranks the seller BSs according to the amount of the shared energy with each of the seller BSs and its price. Assigning a set of buyer BSs to a seller BS can be formulated as a many-to-one matching problem \([34]\). In the many-to-one matching problem, a subset of the buyer BSs in \( B \) is assigned to a seller BS. A buyer BS is assigned to one seller BS at most. In this problem, many-to-one matching is a two-sided matching. The reason is that all involved BSs belong to one of the two sets, sellers and buyers. Matching is defined in \([34]\) as:

**Definition 1**: Let \( S \) be a set of \( \sum_{k=1}^{K} n_k \) sellers and \( B \) be a set of \( \sum_{k=1}^{K} n_k' \) buyers. A matching is a mapping \( \phi \) from the set \( S \cup B \) into the set of all subsets of \( S \cup B \) such that: (1) \( |\phi(b_{j'})| \leq 1 \) and \( \phi(b_{j'}) \subseteq S \cup B \) for all \( b_{j'} \in B \), (2) \( \phi(s_i) \subseteq 2^S \) for all \( s_i \in S \), and (3) \( \phi(b_{j'}) \subseteq s_k \) if and only if \( b_{j'} \) is in \( \phi(s_k) \) for all \( s_k \in S \) and for all \( b_{j'} \in B \).

In the above definition, \( |A| \) denotes the cardinality of the set \( A \). Moreover, \( \phi(b_{j'}) = \emptyset \) means that the buyer \( b_{j'} \) is not successful in finding any seller BS and it is not matched.

B. Utility-Based Preferences

Each buyer BS \( b_{j'} \), the \( j^\text{th} \) BS in tier \( k \), has a strict, transitive, and complete preference relation \( \succ b_{j'} \) over the members in \( S \). The same argument holds for the preference of the members in \( S \) which is denoted by \( \succ s_k \) over members of the set \( S \). In order to obtain the preferences of each BS, the utility functions introduced in the previous section are used. Based on the derived price of the shared energy between a seller BS and a buyer BS, the buyer BS searches for a seller BS that makes its utility maximum compared to other BSs. Therefore, \( \forall s_1, s_2 \in S \) and \( \forall b_{j'} \in B \), a buyer preference is \( s_1 \) over \( s_2 \) as

\[ U^{b_{j'}}(\eta(E_{T_1}), t) > U^{b_{j'}}(\eta(E_{T_2}), t) \Leftrightarrow s_1 \succ b_{j'} s_2, \]  

(15)

where \( \eta(E_{T_i}) \) is the shared energy price between the seller \( s_i \) and the buyer \( b_{j'} \). According to its preference, a buyer BS ranks
TABLE II. SUMMARY OF THE PROPOSED MATCHING ALGORITHM

**Phase 1 - Initialization:**
Each BS broadcasts its tier, the amount of energy in its battery and the amount of energy it wants to sell or to buy to other BSs. Sellers are stored in the set $S$. Buyers are stored in the set $B$.

**Phase 2 - Matching Buyer BSs to Seller BSs**
repeat:
Each buyer BS finds the amount of the shared energy and its price with every seller BSs.
Each buyer BS ranks seller BSs according to its preference.
Each buyer BS sends a request to the first ranked seller BS.
Each seller BS ranks requested BSs with respect to its preference.
repeat:
A seller is chosen and it accepts the first ranked requested buyer.
The extra amount of energy in the seller is updated.
If a buyer BS buys its needed energy, it is removed from $S$.
If a seller BS sells all its extra energy, it is removed from $S$.
The CCS updates credit accounts according to submitted trade characteristics.
BSs in sets $S$ and $B$ broadcast their tiers, the amount of energy in their batteries and the amount of energy they want to sell or to buy.
until the set $S$ is empty or the set $B$ is empty.

**Phase 3 - The Energy Distribution**
The energy distribution is done according to saved energy trades.

C. The Proposed Matching-Game-Based Algorithm

In the first phase of the algorithm, each BS broadcasts its tier, battery level and the extra or needed energy, and BSs are divided into two groups, i.e., sellers and buyers. In phase 2, buyer BSs rank seller BSs according to their preference relations. Next, each buyer BS sends a request to the first ranked seller BS. Seller BSs receive a number of requests and they rank requested BSs according to their preferences due to the fact that seller BSs can calculate the price of the shared energy according to the broadcasting. After ranking requested BSs, a random seller accepts $l$ top ranked requested buyer BSs such that the total requested shared energy with $l$ buyers is less than or equal to its extra stored energy and the total requested shared energy with $l + 1$ buyers is more than its extra stored energy. If a seller depletes its extra energy by selling, it is removed from $S$. Similarly, if a buyer BS compensates its energy deficit, it is removed from $B$. Successful energy trades are saved and the CCS updates credit accounts. BSs in sets $S$ and $B$ broadcast the amount of energy in their batteries and the amount of energy they want to sell or to buy as well as their tiers, and the phase 2 is iterated until $S = \emptyset$ or $B = \emptyset$.

In the phase 3 of the algorithm, the extra harvested energy is distributed according to saved energy trades. The summary of the proposed algorithm is given in Table II. A buyer BS cannot request more than its energy deficit. If it sends a request more than its needed energy and a seller BS accepts its request, the buyer BS reduces the amount of the extra energy for sell. This action of a buyer BS results in additional number of iterations.

Therefore, the seller BS is prevented to help other buyer BSs. By using many-to-one matching, a subset of buyer BSs is assigned to one seller BS. In this case, a BS sells its energy to multiple buyer BSs. If the energy deficit of the buyer BS is not compensated completely, it can send request to another seller BS in the following iteration. Thus, a BS can trade energy with several BSs. Although the energy trading framework, including BSs utility functions, transaction fees, energy trade prices, and preference relations are unique to our paper, we used matching theory, as a mathematical tool, to assign seller BSs to buyers. We use the following definition in the matching theory to prove that the algorithm given in Table II converges.

**Definition 2:** Suppose that $M(S, B)$ is the set of all possible matchings. A many-to-one matching is blocked if $\exists \phi' \in M(S, B), s_k^i \in S$ and $b_k' \in B$ s.t. $\phi'(b_k') \geq b_k'$ and $\phi(s_k^i) \geq s_k^i$. Many-to-one matching is stable if there is no subset of buyer BSs and a seller BS by which the matching is blocked.

Since buyer BSs request seller BSs according to their preference relations, seller BSs are chosen by buyer BSs and seller BSs accept request according to their preferences, the matching cannot be blocked. Buyers buy energy from those BSs that accept their requests. In other words, when the request of a buyer BS is accepted by a seller BS, the buyer BS does not request other seller BSs. If the energy deficit of the buyer BS is not compensated completely, it can send request to another seller BS in the following iteration. Seller BSs also sell their energy according to their preference relations. In this way, energy trades take place and the energy deficit of BSs are compensated gradually. Therefore, the matching is stable, and in each iteration, a number of seller and buyer BSs are removed from $S$ and $B$. Since the cardinalities of $S$ and $B$ are not infinite, these cardinalities decrease gradually as the proposed algorithm iterates and after a number of iterations one or both of them become zero. Consequently, the proposed algorithm converges.

IV. ENERGY TRADING SCHEME AMONG BSs USING A DOUBLE-AUCTION-BASED ALGORITHM

Auctions and double auctions are used commonly in resource allocation problems (see, e.g., [17], [21]–[23]). Each double auction consists of two groups of players, sellers and buyers. The distribution of the extra harvested energy from a number of seller BSs among a number of buyer BSs with energy deficit can be done by using a double auction mechanism. In this section, we use a double auction to determine energy trades prices, seller and buyer BSs that can trade, and the amount of energy each BS trades. Each BS analyzes its battery capacity, battery level and the required energy to serve connected users. In this section, we define different energy valuations for BSs to reflect the fact that the harvested energy and the demanded energy are correlated in time to the proposed approach. We demonstrate that a BS best response is to report its valuations of an energy portion as a bid to the auctioneer. The auctioneer, based on the proposed bids, enters a number of seller and buyer BSs to the Groupwise-Price double auction proposed in [38]. Buyer BSs are motivated to compensate their energy deficit by the extra energy of seller BSs.
By using an optimization embodied in the double-auction-based algorithm, we assign close BSs to each other to trade energy. We show that the proposed algorithm satisfies truthfulness, individual rationalities and budget balance, consequently, keeps BSs motivated to participate in energy trades.

A. Value of a Portion of Energy for Each BS

Consider that one Joule is divided into $\delta$ equal indivisible portions. A needed energy portion has a value for a buyer BS as it helps serving connected users. An extra stored energy has a value for a seller BS since it may become needed in coming time slots. BSs find the number of energy portions they want to sell or to buy based on $\rho_i^{s}(t)$ and $\rho_i^{b}(t)$. Energy portion values are found according to the battery levels so as to enable the proposed algorithm to prevent energy wasting. To differentiate the seller BSs with respect to the amounts of the extra stored energy, and to differentiate the buyer BSs with respect to the amounts of the needed energy, each seller or buyer is provided with a variable $u(t)$ in the time slot $t$. The variable $u(t)$ determines the type of BS and it is a private information of the BS.

We define the type of the seller $s_k^i, \forall s_k^i \in S, \forall k \in \{1, 2, \ldots, K\}$ as

$$u^{s_k^i}(t) = \frac{c_k - \rho_i^{s}(t)}{c_k}.$$  

(17)

It is observed that the type of each seller BS is derived from its battery capacity and its extra harvested energy. The type of the buyer BS $b_{k'}^v, \forall b_{k'}^v \in B, \forall k' \in \{1, 2, \ldots, K\}$ is defined as

$$u^{b_{k'}^v}(t) = \frac{\rho_i^{b}(t)}{T \rho_i^{b}(t)}.$$  

(18)

The type of each buyer BS is derived from its harvested energy and the demanded energy. The battery capacity and the extra harvested or the needed energy of each BS is different from the other BSs. Therefore, the type of each BSs is different from types of other BSs. We classify BSs into tiers to find seller BSs types. We define different values for portions of the extra or the needed energy for a BS to reflect the fact that both the harvested energy by a BS and the demanded energy from a BS are correlated in time to the bidding process. We use the defined type of a BS to find the value of a portion of energy. The value of a portion of energy for each BS is a non-decreasing function of its type.

The seller BS $s_k^i$ contains $\lfloor \rho_i^{s}(t) \delta \rfloor$ extra portions of energy in its battery. We define the value of the $q^{th}$ portion of the extra stored energy of a seller BS $s_k^i$ as

$$v_q^{s_k^i}(u^{s_k^i}(t)) = \left[ u^{s_k^i}(t) + \frac{q}{S c_k} \right] \frac{\delta}{\delta},$$  

(19)

where $\varphi$ is a design parameter between 0 and 0.5 that is used to control the values of energy portions for each BS. The values of energy portions increases with step size $\delta \cdot c_k$ sequentially as the energy portion number increases. When the BS $s_k^i$ has $\lfloor \rho_i^{s}(t) \delta \rfloor$ extra energy portions, it is observed from (19) that the infimum of an energy portion value for the seller BS is $\frac{\delta}{S c_k}$ and its supremum is $\frac{\delta}{\delta}$. The value of the $q^{th}$ portion of energy for the buyer BS $b_{k'}^v$ is defined as

$$v_q^{b_{k'}^v}(u^{b_{k'}^v}(t)) = \left[ u^{b_{k'}^v}(t) \varphi + (1 - \varphi) - \frac{q \varphi}{T \rho_i^{b}(t) \delta} \right] \frac{\delta}{\delta},$$  

(20)

As the BS $b_{k'}^v$ needs $\lfloor \rho_i^{b}(t) \delta \rfloor$ energy portions, the supremum of an energy portion value for a buyer is $\left[ 1 - \frac{\varphi}{T \rho_i^{b}(t) \delta} \right] \frac{\delta}{\delta}$ which is less than $\frac{\delta}{\delta}$ and the infimum of an energy portion value is $\frac{\delta}{\delta} - \frac{q \varphi}{T \rho_i^{b}(t) \delta}$. The values of energy portions decrease with step size $\frac{\delta}{\delta} - \frac{q \varphi}{T \rho_i^{b}(t) \delta}$ as the energy portion number increases.

Finally, the value of the last portion of the required energy for buyer BSs gets close to $\frac{\delta}{\delta} - \frac{q \varphi}{T \rho_i^{b}(t) \delta}$. While $\varphi \in (0, 0.5)$, we have $\frac{\delta}{\delta} < \frac{\delta}{\delta} - \frac{q \varphi}{T \rho_i^{b}(t) \delta}$ and hence, the value of a needed energy portion for a buyer is always greater than or equal to the value of an extra energy portion for a seller. As the extra harvested energy of the seller BSs increases and gets close to the battery capacity, $u^{s_k^i}(t)$ decreases. Thus, $v_q^{s_k^i}(u^{s_k^i}(t))$ decreases. In the following subsection, we show that as $v_q^{s_k^i}(u^{s_k^i}(t))$ decreases, the extra energy portions of the BS $s_k^i$ are sold quickly which prevents the possible energy waste in coming time slots. As tradable energy portion number, $q$, increases, the energy portion value increases. We show that this feature enables selling energy portions of other sellers with high extra energy which prevents the waste of energy in their batteries. Furthermore, when the energy deficit increases, the type of the buyer BS, operating in arbitrary tier, increases. As $u^{b_{k'}^v}(t)$ increases, $v_q^{b_{k'}^v}(u^{b_{k'}^v}(t))$ increases and we show that the needed energy portions of $b_{k'}^v$ are bought quickly. The first priority for buying energy is assigned to the buyer BSs with higher types, thus, the fairness of the energy distribution increases. As $q$ increases, the value of an energy portion for a buyer decreases. It is shown this feature enables buyers with high energy deficit to buy energy and increases the fairness. It is seen that to find energy values for a BS, one need to know battery capacity of the BS; that is why BSs are classified into different tiers. Fig. [1] depicts the value of energy portions for a macrocell when $\delta = 1, T = 1$ second and $\varphi = 0.4$. The values of the extra energy portions of a seller macrocell when $\rho_i^{s}(t) = 8$ Joule are depicted. Additionally, the values of the needed energy portions of a buyer macrocell when $\rho_i^{b}(t) = 10$ Joule are depicted. As the number of the extra energy portion increases, the values of the extra energy portions increases.

B. The Proposed Double-Auction-Based Algorithm

Here, we introduce an algorithm that participates BSs in a double auction in order to trade energy. This algorithm includes three phases that are performed successively at the beginning of each time slot. The seller and buyer BSs are considered to be players that their strategies (their actions) are reporting bids to the auctioneer, simultaneously. The auctioneer and control center of the network is the CCS in the proposed algorithm. The cloud radio access network (CRAN), is a new technology used for cellular networks architecture to develop a promising control center [39]. This is a candidate place for running CCS. We use defined energy values, and the Groupwise-Price double auction introduced in [38] to design a double-auction-based algorithm. Consider the seller BS $s_k^i$ trades $\chi^{s_k^i}(t)$ energy portions with buyer BSs and it receives $w^{s_k^i}(t)$ revenue for the $q^{th}$ energy portion sold in the double auction. The vector of the received revenues for trading $\chi^{s_k^i}(t)$ energy portions is $\mathbf{w}^{s_k^i}(t)$. The utility function of the seller BS $s_k^i, \forall s_k^i \in S, \forall k \in \{1, 2, \ldots, K\}$ is

$$U^{s_k^i}(\chi^{s_k^i}(t), \mathbf{w}^{s_k^i}(t)) = \frac{\zeta T p^{s_k^i}(t) + \sum_{q=1}^{n^{s_k^i}(t)} w^{s_k^i}(t) - v_q^{s_k^i}(u^{s_k^i}(t))}{S}.$$  

(21)

The first term of the above utility is earned from the connected users and the second term is earned from selling the extra stored energy portions.
energy. The buyer BS $b^t_k$, $\forall b^t_k \in B$, $\forall k' \in \{1, 2, \ldots, K\}$, trades $\chi^t_{k'}(t)$ energy portions with seller BSs and it pays $w^t_{k'}(t)$ for the $\frac{q}{K}$ needed energy portion. The vector of paid revenues for trading $\chi_{k'}^t(t)$ energy portions is $\mathbf{w}^t_{k'}(t)$. The buyer BS utility function is given in (22). In (22), $\rho^t_{k'}(t) - \chi^t_{k'}(t)$ is the amount of energy that the buyer BS $b^t_k$, cannot obtain from other BSs and buys it from the non-renewable source at the price of $\left(\frac{\rho^t_{k'}(t)}{\delta} - \chi^t_{k'}(t)\right)\psi$. Additionally, $s_q,b^t_k$ is the seller BS that the buyer $b^t_k$ buys its $q$th needed energy portion from it. We assume that each BS reports the real valuation of a portion of energy as the corresponding bid to that portion. We demonstrate later that this is the best response. The tier of a BS, the amount of energy in the battery and the amount of energy it wants to sell or to buy are required by the CCS to find the type and bids of each BS. In other words, reporting these information is equal to reporting the type and bids by a BS. Those information are reported to the CCS at the beginning of each time slot. The BSs are divided into two groups, i.e., seller BSs and buyer BSs, to setup a double auction. Assume that the bid of the BS $s^k_q$ for the $q$th extra energy portion is denoted by $b^t_{s^k_q}$ and the bid of the BS $b^t_k$, for the $q$th needed energy portion is denoted by $b^t_{b^t_k}$. Thus, based on the above assumption, we have $s^t_{b^t_q} = v^t_q\left(u^t_q(t)\right)$ and $b^t_{b^t_k} = v^t_{b^t_k}\left(u^t_{b^t_k}(t)\right)$. The number of reported bids to the CCS from the seller BS $s^k_q$ is $\left|\delta \rho^s_k(t)\right|$. Since there are $n_k$ seller BSs in tier $k$, the number of reported bids by seller BSs collected by the CCS is $\sum_{k=1}^K \sum_{s=1}^{n_k} \left|\delta \rho^s_k(t)\right|$. The CCS operates the Groupwise-Price double auction. The CCS, regardless of their indices, orders and numbers bids. The CCS orders and numbers the bids from seller BSs in an increasing order as follows

$$s^t(1) \leq \cdots \leq s^t\left(\sum_{s=1}^{n_k} \left|\delta \rho^s_k(t)\right|\right).$$

Similarly, it orders the buyers bids in a decreasing order as

$$b^t(1) \geq \cdots \geq b^t\left(\sum_{t=1}^T \sum_{s=1}^{n_k} \left|\delta \rho^s_k(t)\right|\right).$$

In the second case of the algorithm, a price $y$ is considered by the CCS that does not depend on BSs types to prevent BSs from misreporting their types, and thus, the truthfulness is resulted. The price is random and it follows any arbitrary distribution between 0 and $\frac{1}{2}$ such that the probability of at least one point among $\frac{x}{2}$ and $\frac{1-x}{2}$ is non-zero. This feature of $y$ is used to prove that the algorithm converges. For each bid like $v^t_q\left(u^t_q(t)\right)$ which is less than or equal to $y$, the CCS saves an ordered pair, $(s^k_q,q)$, that its first element determines the seller BS that the bid belongs to. The second element specifies the number of the extra energy portion that the bid is made for it. Ordered pairs are saved in the set $S_o$ as

$$S_o = \{ (s^k_q,q) | v^t_q\left(u^t_q(t)\right) \leq y \}. $$

The above set is used in assigning seller BSs to buyer BSs and determining the price that is paid to each seller BS. Likewise, for each bid like $v^t_{b^t_k}\left(u^t_{b^t_k}(t)\right)$ which is greater than or equal to $y$, the CCS reserves an ordered pair, $(b^t_k,q)$, that its first element determines the buyer BS that the bid belongs to. The second element specifies the needed energy portion number that the bid is made for it. Ordered pairs are saved in $B_o$ as follows

$$B_o = \{ (b^t_k,q) | v^t_{b^t_k}\left(u^t_{b^t_k}(t)\right) \geq y \}. $$

In order to match the supply and demand, the minimum of the cardinalities of the sets $S_o$ and $B_o$ is considered and called $\kappa$. Regarding orders in (23), corresponding ordered pairs to the first $\kappa$ lowest bids are reserved in the set $S^*_o$. In the same way, corresponding ordered pairs to the first $\kappa$ highest bids in (24) are reserved in the set $B^*_o$. In Fig. [1] $y = 0.36 \zeta$, and thus, the cardinality of sets $S_o$ and $B_o$ are six and ten, respectively. The minimum cardinality, $\kappa$, is six. Consequently, corresponding ordered pairs to the six top bids of the buyer are stored in $B^*_o$ and corresponding ordered pairs to the six lowest bids of the seller BS are reserved $S^*_o$. Six energy portions are traded. It is seen that since $y \notin (\frac{0.36 \zeta}{2}, \frac{(1-0.36) \zeta}{2})$, this random price, $y$, cannot enter all eight extra stored energy portions to the double auction. In this example, six needed energy portions can be compensated by the available extra energy. To find the price that each buyer BS in $B^*_o$ has to pay and the revenue paid to each seller BS in $S^*_o$, the Groupwise-Price double auction mechanism is used. The CCS finds the type of each seller BS in $S^*_o$. Consider that $u^t_{s^k_q}(t)$ denotes the vector of all seller BSs types, excluding $u^t_{s^k_q}(t)$. A variable is assigned to each seller BS, denoted by $z^s_k(u^t_{s^k_q}(t))$, which is greater than or equal to its type. By using the value of energy portions formula given in (19), the CCS obtains the variable $z^s_k(u^t_{s^k_q}(t))$ for all $(s^k_q,q) \in S^*_o$ to calculate the revenue that the seller BS $s^k_q$ receives, accordingly. This variable is the supremum of the possible quantities for the seller BS type that if the BS takes it, the ordered pair, $(s^k_q,q)$, which corresponds to the bid $v^t_q\left(u^t_q(t)\right)$, remains in $S^*_o$. In other words,

$$z^s_k(u^t_{s^k_q}(t)) = \sup \left\{ z^s_k((s^k_q,q) \in S^*_o | (z^s_k(u^t_{s^k_q}(t)) \left( u^t_{s^k_q}(t) \right)) \right\}.$$
optimization is solved to assign seller BSs to buyer BSs

\[
\min_{I_s, b_j} \sum_{i=1}^{|S_a^*|} \sum_{j=1}^{B^*_a} g^s_i, b_j \ I_s, b_j \\
\text{s.t.} \quad \sum_{i=1}^{|S_a^*|} I_s, b_j = 1, \quad \forall s_i \in S_a^*, \quad \sum_{j=1}^{B^*_a} I_s, b_j = 1, \quad \forall b_j \in B^*_a,
\]

(29)

where \(g^s_i, b_j\) is the distance among the seller \(s_i\) and the buyer \(b_j\). In the above optimization problem, \(I_s, b_j\) is a variable that seller and buyer BSs are assigned by it. If \(I_s, b_j\) trades an energy portion with \(b_j\) and \(I_s, b_j = 0\) otherwise. The first constraint captures the fact that each extra energy portion can be sold only once. The second constraint follows from the fact that each needed energy portion can be bought only once. The optimization problem in (29) is the traveling salesman problem [40, p. 276] which is demonstrated to be NP-Hard. Since the proposed algorithm is used for real time applications, the following approximate solution is used.

**Approximate solution:** The nearest neighbor algorithm is used to approximate the solution of the optimization in (29). This algorithm quickly gives an efficient solution [41]. The CCS finds the buyer BS that proposes the highest bid and assigns it to a seller BS in \(S_a^*\) that is the nearest to that buyer BS. The buyer BS that proposes the highest bid, denoted by \(b_j^*\), is found as

\[
b_j^* = \left\{ b_j^* | \forall b_j, b_j^* \in B^*_a, \ u^s_i, b_j (t) \leq u^s_i, b_j^* (t) \right\}. \tag{30}
\]

Remind that each element of the set \(B^*_a\) corresponds to a bid made for an energy portion. In (30), \(q\) and \(q'\) are the number of energy portions that BSs \(b_j\) and \(b_j^*\) require and propose bids to buy them. The seller BS in \(S_a^*\) that is the nearest to the buyer BS \(b_j^*\) is found from the following formula

\[
s_i^* = \left\{ s_i^* | \forall s_i, s_i^* \in S_a^*, \ g^s_i, b_j \leq g^s_i, b_j^* \right\}. \tag{31}
\]

Similarly, each element of the set \(S_a^*\) corresponds to a bid made for an energy portion. Then, the corresponding ordered pairs to the highest bid of buyers and the lowest proposed bid by the nearest seller BS are removed from \(B^*_a\) and \(S_a^*\), respectively. The highest bid of the buyer BS and the lowest bid of the near seller are removed from existing bids at the CCS. In addition, these seller and buyer BSs are removed from \(S_a^*\) and \(B^*_a\), respectively. Similarly, the highest remaining bid is regarded and it is assigned until \(S_a^* = B^*_a = \emptyset\). In this way, the seller and buyer BSs are assigned to each other such that the total smart grid usage is reduced. Energy trades are saved and the CCS updates credit accounts. According to distances among BSs and the amount of extra energy of each seller BS, a buyer BS can trade energy with several BSs. Similarly, a seller BS can trade energy with several buyer BSs according to their required energy and distances from the seller BS.

Bids corresponding to successful energy trades are removed from existing bids in the CCS. If there is no bid for selling an energy portion or there is no bid for buying an energy portion, the algorithm begins the third phase. Otherwise, a
new random price is determined and new sets $S^*_s$ and $B^*_o$ are formed. BSs trade energy in the same way explained above. Regarding a random price and trading accordingly is repeated until either there is no bid to sell an energy portion or there is no bid to buy an energy portion. The third phase is that energy distribution is done by the smart grid according to saved energy trades. The summary of the proposed algorithm, which is run at the beginning of each time slot, is given in Table III. As the supremum of the value of a portion of energy for a buyer BS is $\frac{\zeta}{2}$, the price that a buyer BS pays for a portion of energy in the double auction is at most $\frac{\zeta}{2}$. Since $\zeta + \max_{g^k,b^k} \Gamma(g^k,b^k) < \psi$, we find that $\frac{\zeta + \Gamma(g^k,b^k)}{2} < \frac{\psi}{2}$. Therefore, the buyer BS has always incentive to compensate its energy deficit by the extra harvested energy of other BSs rather than the non-renewable energy. BSs are motivated to trade energy in the double auction. It is observed that as each BS reports its tier, the amount of stored energy in its battery, and the amount of energy it want to buy or to sell to the auctioneer. Consequently, if there exist $\zeta$ BSs in the network, the number of information broadcasting is $\zeta$, however, it is $\zeta (\zeta - 1)$ for the previous algorithm. Thus, the amount of distributed BSs information in the network reduces considerably compared to decentralized algorithm given in the last section.

**Proposition 1:** The proposed double-auction-based algorithm converges.

**Proof:** As explained in Table III, the proposed algorithm is iterative. The number of energy portions sold in each iteration depends on $\varphi$. Since we assume that $\varphi \in (0, 0.5)$, and $\frac{\zeta}{\zeta} < \frac{1}{\zeta^2}$, a $y$ between $\frac{\zeta}{\zeta}$ and $\frac{1}{\zeta^2}$ can compensate all the needed energy portions or sell all extra energy portions. However, other $y$ quantities assign a part of the demanded energy portions to the extra stored energy portions. As $y$ is random and the probability of at least one point among $\frac{\zeta}{\zeta}$ and $\frac{1}{\zeta^2}$ is non-zero, there exists a price that can compensate all the needed energy portions or sell all extra energy portions. In each iteration, a number of surplus energy portions are sold and a number of needed energy portions are bought. Since the number of the extra stored energy portions and the required energy portions are not infinite, the algorithm converges.

We investigate some properties of the proposed algorithm. Definitions of properties investigated in following properties are given in [25].

**Proposition 2:** The double-auction-based algorithm maintains individual rationalities of BSs.

**Proof:** To show this property, we note that each seller BS like $s^k$ contains $\delta \rho^{s^k}(t)$ energy portions to sell and each buyer like $b^k_o$ is intended to buy $\delta \rho^{b^k_o}(t)$ energy portions. If a seller BS cannot sell any of its extra energy portions, it does not gain any negative profit from the double auction. In the same way, if a buyer BS cannot buy any of its needed energy portions, it does not gain any negative profit from the double auction. Now, we show that any sold energy portion of a seller BS provides non-negative profit to the BS. The BS $s^k$ bids $\psi^k_s(u^k)$ for its $q^o$ extra stored energy portion. The payment to $s^k$ for the $q^o$ portion of energy is $\psi^k_s \left( z_q^k \left( u^k \right) \right)$ which is the supremum of the possible quantities for $\psi^k_s$ that keeps $(s^k, q) \in S^*_s$. As the supremum of the set of possible bids for a portion of energy is greater than or equal to the set elements, the revenue that the seller receives is greater than or equal to the energy portion valuation, and thus, the profit gained by the seller is non-negative. Likewise, $\psi^k_o \left( b^k_o \left( u^k \right) \right)$ is the infimum of the set of possible quantities for $\psi^k_o$ that keeps $(b^k_o, q) \in B^*_o$. Since the infimum of the set of possible quantities for $\psi^k_o$ that keeps $(b^k_o, q) \in B^*_o$ is less than or equal to its elements, the payment is less than or equal to the energy valuation for the buyer, and thus, the profit gained by the buyer is non-negative. It is observed that it is not possible for a seller or a buyer BS to gain negative profit. Hence, the algorithm maintains individual rationalities of BSs.

**Proposition 3:** The double-auction-based algorithm is budget balance.

**Proof:** It is observed from Table III that each seller or buyer BS that joins the double auction is reserved in an ordered pair in $S^*_s$ or $B^*_o$, respectively. Remind that the corresponding ordered pair to a bid made to sell a portion of energy is stored in $S_o$, only if the bid is less than or equal to the random price $y$. Since the money that a seller BS receives for selling a portion of energy is the supremum of a bid quantity by which the corresponding ordered pair to the bid remains in $S^*_s$, the amount of money that a seller BS receives is less than or equal to $y$. Similarly, the corresponding ordered pair to a bid made to buy a portion of energy is stored in $B_o$ only if the bid is greater than or equal to the random price $y$. Since the money that a BS pays for buying a portion of energy is the supremum of a bid quantity by which the ordered pair remains in $B^*_o$, the money that a BS pays is greater than or equal to $y$. Therefore, in each iteration, buyers pay greater than or equal to $y$, and sellers receive less than or equal to $y$. Consequently, a part of the paid money by the buyers is remained for the auctioneer. This part is greater than or equal

<table>
<thead>
<tr>
<th>Phase 3 - Energy Distribution</th>
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<tr>
<td>Energy distribution is done according to saved energy trades.</td>
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to zero. In other words, the algorithm is budget balance.

Proposition 4: The double-auction-based algorithm is feasible, non-wasteful and competitive.

Proof: As the algorithm terminates when the extra stored energy depletes, the algorithm is feasible. In other word, the algorithm does not culminates in selling more energy than the extra stored energy in the network. Since the algorithm continues to compensate energy deficit until there exists a seller and a buyer BS, the proposed algorithm is non-wasteful and it compensates energy deficit of the network of BSs as much as possible with the existing extra stored energy. The proposed algorithm is competitive due to the fact that in each iteration, the amount of bought energy portions and sold energy portions are the same. In other words, \( \sum_{k=1}^{K} \sum_{i=1}^{n_k} x^k_i(t) = \sum_{k'=1}^{K} \sum_{j=1}^{n_{k'}} y^{k'}(t) \).

Proposition 5: The double-auction-based algorithm is truthfulness.

Proof: To demonstrate this fact, we show that seller and buyer BSs cannot gain more profit by misreporting their tiers, battery levels or the amount of extra or needed energy. Assume that a seller BS misreports its tier, battery level or the demanded energy. In this scenario, two cases are possible. First, assume that the misreported values result in a bid under the real value of an energy portion for the bidder, i.e., \( s b_{q,j}^k < v_q^k(u^{s^k}(t)) \). In this case, three occasions are possible. The first occasion is that the bidder fails to join the set \( S^*_o \) by under bidding and it fails by bidding the true energy portion value. Under bidding does not increase the bidder profit in this occasion. The second occasion is that by bidding the energy portion value, the seller BS joins the set \( S^*_o \) and it joins the set \( S^*_o \) as well by under bidding. Since the revenue it obtains depends on the bids of other seller BSs and the random price, under bidding cannot change the revenue it obtains. The third occasion is that by reporting the real value of a portion of energy, it cannot join the set \( S^*_o \). However, by under bidding, it joins the set \( S^*_o \). In this occasion, the value of the portion of energy is more than the revenue it receives from the double auction. In other words, the revenue it obtains is \( v_q^k(u^{s^k}(t)) \), and the value of the energy portion is \( v_q^k(u^{s^k}(t)) > v_q^k(z_q^k(u^{s^k}(t))) \). Therefore, under bidding does not change the gained profit or it gains a negative profit. In the second case, a seller reports a bid for a portion of energy more than its real value, i.e., \( s b_{q,j}^k > v_q^k(u^{s^k}(t)) \). In this case, three occasions are possible. In the first occasion, reporting the true value leads to joining the set \( S^*_o \) and gaining a non-negative profit, while over bidding leads to losing to join \( S^*_o \) as well by under bidding. Since the money a buyer BS pays to buy a portion of energy depends on the bids of the other BSs and the random price, its gained profit does not change by over bidding. Considering above cases, it is seen that the buyer BSs do not have incentive to misreport battery level or the needed energy in each iteration of the algorithm. Therefore, seller BSs and buyer BSs report their real tier, battery levels and the extra or the needed energy, and the proposed algorithm is truthful.

Regarding above cases, it is observed that BSs have no incentive to change their actions, which is bidding for energy portions, and the algorithm reaches the Nash equilibrium. The best response of each player, i.e., seller or buyer, is to report the valuations of energy portions as bids since the BS cannot gain more profit when it does not do this. As BSs bids are found from numbers of BSs tiers, their stored energy and the energy they want to sell or to buy by the auctioneer, therefore, the truthfulness of the algorithm guarantees that the algorithm runs with the correct information of BSs energy status. Individual rationalities and budget balance ensure that non-cooperative BSs and the auctioneer are motivated to participate in energy trades, respectively. It is straightforward to demonstrate that the matching-game-based algorithm is non-wasteful in the same way proved for the double-auction-based algorithm. As both algorithms are non-wasteful, they compensate BSs energy shortage as much as possible with the available extra harvested energy in each time slot. Hence, the non-renewable energy consumption cannot be reduced further as long as harvested and demanded energy are stochastic. The termination point of both algorithms are the same. In other words, as long as there exist a BS with energy deficit and a BS with surplus harvested energy, both algorithms compensate energy shortage of BSs with the available surplus energy in the network as much as possible. As a result, the proposed algorithms cannot help each other to reduce the non-renewable energy consumption to a greater extent. Regardless of sellers decisions about choosing buyer BSs, the amount of traded energy in both of the algorithm is constant and it is the minimum of the extra available energy stored in the network and the required energy of
BSs, $\min\left\{\sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \sum_{k'=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t)\right\}$. In both algorithms, we use variable energy pricing to reduce the waste of energy in BSs batteries. High harvested energy in successive time slots results in the waste of energy due to the limited battery capacity of a BS when the energy trading is not enabled. When the extra energy of a seller BS increases, the values of the extra energy portions decrease. As the lower bids to sell energy portions join $S^*_2$ with more probability, the energy portions correspond to lower bids are sold quickly which prevents the possible energy waste in next time slots. On the other side, when the energy deficit is high, the value of a needed portion of energy increases close to $\frac{1}{2}$. As the value of a portion of the needed energy increases, the bid corresponding to that energy portion increases. Therefore, the chance of joining $B^*_S$ and having a successful trade with seller BSs increases. BSs with high energy deficit buy energy quickly that increases the fairness of the energy distribution. To satisfy these properties, the values of energy portions are defined as done in [19] and [20].

Consider the case that the total required energy of BSs in the network is less than the total surplus available energy at BSs. In this case, the surplus energy will be used to compensate the energy deficit of BSs in the network when each of the proposed algorithms is used. When a BS buys more energy than its need, it prevents at least one BS with energy deficit from compensating its required energy. In this case, those BSs with energy deficit buy non-renewable energy, and thus, the non-renewable energy consumption increases. This is the reason that we avoid BSs from buying more energy than their required energy. In addition, when a BS buys more energy than its required amount, it has to store the green energy which is demonstrated to be disadvantageous action [42].

Here, we investigate the complexities of both of the proposed approaches, the matching-game-based algorithm and the double-auction-based algorithm. In the matching-game-based approach, each BS reports its tier, its battery level, and its extra or needed energy to other BSs. Then, each buyer BS finds prices of energy trades with each of the seller BSs. The computational complexity of finding prices is $O(|S||B|)$. Consider that the algorithm converges in $L$ steps. Assume that in the second iteration $|S_2|$ sellers and $|B_2|$ buyers are left. In the last iteration, there are $|S_L|$ sellers and $|B_L|$ buyers. Thus, the computational complexity is $O(3|S| + 3|B| + |S||B| + \sum_{z=2}^{L} |S_z||B_z|)$. The time complexity is also $O(n^2)$.

Moreover, we investigate the complexities of the double-auction-based approach. Each seller BS reports its tier and its surplus energy. Then, the control center computes the bids corresponding to $\sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \delta$ extra energy portions of sellers. Thus, the computational complexity of finding seller BSs bids is $O(2|B| + \sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \delta)$. Each buyer BS reports its tier, its required energy and its battery level in that time slot. The control center computes the bids corresponding to $\sum_{k=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t) \delta$ required energy portions of buyers. Thus, the computational complexity of finding buyer BSs bids is $O(3|B| + \sum_{k=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t) \delta)$. Assume that $y \in \{\phi, \frac{\phi}{2}, (1-\phi)\frac{\phi}{2}\}$. In the next step of the algorithm, the trading saleman problem is solved by using the nearest neighbor algorithm. The number of traded energy portions is $\min\left(\sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \delta, \sum_{k=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t) \delta \right)$. Thus, $\min\left(\sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \delta, \sum_{k=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t) \delta \right)$ surplus energy portions are sold and $\min\left(\sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \delta, \sum_{k=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t) \delta \right)$ energy portions are bought. According to [43], the complexity of the nearest neighbor algorithm to solve the optimization problem in (29) is $O\left(\left(\min\left(\sum_{k=1}^{K} \sum_{i=1}^{n_k} \rho^i_k(t) \delta, \sum_{k=1}^{K} \sum_{j=1}^{n_{k'}} \rho^{j\prime}_{k'}(t) \delta \right)\right)^2\right)$. Thus, the complexity of this approach is given in (32). The time complexity of both of the approaches is $O(n^2)$ which is similar to the proposed approach in [17]. A closed-form for the lower bound of the expected obtained revenue from selling an energy portion is found in [44].

In the matching game-based-algorithm, when the energy loss during the energy transfer is taken into account, the buyer BSs compute the amount of energy that they receive from each seller BSs. Furthermore, they compute their utility functions based on the amount of energy that they receive. In other words, the utility functions of buyer BSs are modified as follows

$$U^{b\prime}_{k}(\eta(E_T(t), t)) = \zeta T p^{b\prime}_k(t) - \eta(E_T(t)) - E_N(t) \psi - E_T(t) \Gamma(g^{s_k,b_l})$$

where $E_N(t) = p^{b\prime}_k(t) - E_T(t) \Omega_k^{s_k,b_l}$ is the amount of bought non-renewable energy and $\Omega_k^{s_k,b_l}$ is the efficiency of energy delivery among the seller BS $s_k$, and the buyer BS $b_l$, which is an increasing function of the distance between BSs. As the buyer BS pays the price for buying $E_T(t)$ Joule and it receives $E_T(t) \Omega_k^{s_k,b_l}$ Joule, thus, it prefers to buy renewable energy from nearby BSs. The battery level of each seller BS is updated as

$$e^{s_k}(t + 1) = \min\left(\max\left\{e^{s_k}(t) - T p^{s_k}(t), 0\right\} + \mu^{s_k}(t) - E_T(t + 1), c_F\right)$$

and the battery level of each buyer BS is updated by using (35). As long as the utility function (6) is substituted by the above utility function and energy in batteries of BSs is updated as done above, the matching game-based-algorithm performs in the presence of energy loss during delivery. The non-renewable energy consumption increases in this case compared to the case that there is no loss. The reason is that the lost energy is used to compensate energy deficit of BSs when the energy loss is negligible.

In the double-auction-based algorithm, however, it is more complex to investigate the effect of the energy loss in delivery. The reason is that it is not known before the auction that from which seller BSs a buyer BS buys energy. The strong aspect of the centralized approach is that close BSs are assigned to each other by the optimization problem formulated in (29). Thus, the waste of energy during delivery is reduced compared to the case that the algorithm does not consider the distances among BSs. When the energy loss is taken into account in this approach, the utility function of the buyer BS is modified as (36). When the equation (22) is substituted by the (36), and we use (34) and (35) instead of (2), the double-auction-based algorithm performs in the new environment. Similar to the previous case, a part of renewable is wasted during delivery, thus, the non-renewable energy consumption in the network increases.

V. PERFORMANCE RESULTS

We consider a three-tier small cell network, including macrocells (tier 1), microcells (tier 2) and picocells (tier 3). BSs of tiers are distributed according to PPPs with densities $[1000, 300, 300]$ [m$^{-2}$], respectively, in a 1.35 km $\times$ 1.35 km area. Moreover, users distribution follows a PPP with density $500$ [user/m$^2$]. The maximum transmit power of BSs depend on their
tiers, sorted as [40, 6.3, 1] Watt. We assume $\gamma = 4$, the coverage probability is 0.65, the time slot duration is one second and $\varphi = 0.4$, $\delta = 100$. It is considered that shadowing attenuates transmitted signals and it follows a lognormal distribution with the same mean and variance for all tiers. Additionally, we consider $\vartheta = 0.7$ and $\beta = 0.4$. The harvested energy and the demanded energy are modeled by correlated Gaussian processes. Gaussian processes are correlated by Cholesky decomposition method. Both the harvested energy and the demanded energy at each time slot are correlated by their previous values at the last time slot and two previous time slot. Means of the demanded power of users connected to different tiers during a time slot are [22.5, 5, 0.85] Watt, respectively. The required smart infrastructures to enable two-way power and information delivery in a network are investigated in [13]. BSs in a cellular networks can be considered as energy loads in the smart grid [15], [16], [19]. To enable two-way energy trading among BSs via smart grid, smart infrastructure system is required. The smart infrastructure system includes energy, information, and communication facilities. The smart communication subsystem makes information transmission among nodes possible. Smart management system is also needed which is responsible for providing advanced management and control services [13]. Smart metering as a part of smart information subsystem also plays an important role and controls energy consumption of nodes in smart grid and the data information is sent to the control center of smart grid by it. In the simulations, we assume that the above described infrastructure is installed in the network [13]. Detailed required infrastructure needed to run energy delivery in smart grid is given in [13].

Effects of the proposed matching-game-based and double-auction-based algorithms, and battery capacities of tiers on the consumed non-renewable power are depicted in Fig. 2. The sum of the harvested energy of BSs in the network in all time slots are more than the demanded energy from BSs in the network. Thus, the stored energy in the network is sufficient to serve connected users in all time slots. However, the harvested energy by a BS is not enough in some time slots due to its stochastic nature. By using the proposed matching-game-based and the double-auction-based algorithms, the consumed power from the non-renewable source is reduced considerably. In the initial time slots, larger battery capacity is not influential. The reason is that the extra stored energy is less than the battery capacity. As more time slots are elapsed, the extra stored energy increases, and larger battery capacities become more helpful. The proposed algorithms distribute the extra stored energy among BSs with energy deficit instead of storing the extra energy in batteries. Hence, BSs require smaller batteries to store energy when the algorithms are applied. Applying the algorithms removes the cost of installing large batteries. When the algorithm is not applied, the non-renewable energy consumption increases as the harvested energy is wasted in limited batteries. The termination points of both algorithms are the same and they iterate as long as there exists a BS that its energy shortage is not applied, the non-renewable energy consumption increases. For the rest of simulations, we consider that battery capacity of BSs in different tiers are sorted as [24, 7, 3] Joule. The performance of the proposed algorithms are compared to the performance of the given energy trading scheme in [17] in Fig. 2. Bids of sellers are chosen randomly in [17]. The amount of traded energy and gained profits of storage units are stochastic.

\[
O\left(\min \left(\sum_{k=1}^{K} \sum_{i=1}^{n_k} \left| \rho_i^{k}(t) \right| \delta, \sum_{k'=1}^{K} \sum_{j=1}^{n_{k'}} \left| \rho_j^{k'}(t) \right| \delta \right) \right)^2 + 2 |S| + \sum_{k=1}^{K} \sum_{i=1}^{n_k} \left| \rho_i^{k}(t) \right| \delta + 3 |B| + \sum_{k'=1}^{K} \sum_{j=1}^{n_{k'}} \left| \rho_j^{k'}(t) \right| \delta \right) \]

(32)

\[
e^{b_{k'}(t + 1)} = \min \left\{ \max \left\{ e^{b_{k'}(t)} - T p_{k'}^{b_{k'}}, 0 \right\} - \mu_{k'}^{b_{k'}}, E_T(t + 1), c_k \right\}. \]

(35)

\[
U_{k'}^{b_{k'}}(x_{k'}^{b_{k'}}(t), w_{k'}^{b_{k'}}(t)) = \zeta T p_{k'}^{b_{k'}}(t) \]

\[
+ \sum_{q=1}^{n} \left[ u_{q}^{b_{k'}}(w_{k'}^{b_{k'}}(t)) - u_{q}^{b_{k'}}(x_{k'}^{b_{k'}}(t)) - \Gamma_{q}^{s_{k'}, b_{k'}} / \delta \right] - \left( \rho_i^{b_{k'}}(t) - \sum_{q=1}^{n} x_{q}^{b_{k'}}(t) / \delta \right) \psi. \]

(36)

Fig. 2. Comparison of the non-renewable power consumption in the network with and without applying the matching-game-based (MGB) and the double-auction-based (DAB) algorithms by different battery capacities. The performance of the energy trading scheme in [17] is shown when $c_1 = 24$, $c_2 = 7$ and $c_3 = 3$ Joule. Means of the harvested energy of different tiers are [23.3, 5.8, 1.1] Joule.
The smart grid usage (Joule × meter)

The cumulative utility of a macrocell

Consequently, as the amount of traded energy is stochastic, it fails to compensate energy deficit of BSs thoroughly in most of time slots. In [17], the required energy of each storage unit is not always compensated completely via the surplus energy of other storage units. Thus, the proposed energy trading scheme in [17] results in a greater non-renewable energy consumption as it is seen from Fig. 3. When the battery capacity of energy storage units is limited, to prevent the surplus harvested energy from wasting, it is better to sell the excessive energy quickly. Therefore, to reduce the waste of energy, the priority of selling excessive energy should be assigned to those storage units that their surplus energy is close to their battery capacity. However, as the bids of sellers in [17] are random, it is possible that a storage unit with large amount of surplus energy cannot sell its energy, however, a storage unit with small amount of excessive energy sells its energy. In this case, the waste of energy increases due to the energy overflow in limited battery of the energy storage units. The result shown in Fig. 3 substantiates the given fact in [42] that banking renewable energy is disadvantageous.

The smart grid usage of each algorithm, which equals to sum of the amount of transferred energy times transmission distance of energy trades, is depicted in Fig. 3. It is seen that the double-auction-based algorithm performs better than the matching-game-based algorithm in the smart grid usage reduction due to the fact that an optimization for assigning seller BSs to buyer BSs is embodied in the double-auction-based algorithm. In the matching-game-based algorithm, however, buyer BSs find seller BSs according to their utility functions. Although the smart grid usage cost exists in the utility functions of buyer BSs, it is not the only major factor in finding a seller BS. The other parameters like battery levels of seller BSs affect buyer BSs decision. On the other hand, near BSs are assigned to each other to trade energy by the double-auction-based algorithm that results in more smart grid usage reduction. In Fig. 3, two curves are presented for the double-auction-based algorithm. In the above curve of the double-auction-based algorithm performance, \( y \in (0, \frac{1}{2}) \). In the below curve of double-auction-based algorithm performance, \( y \in (0.4, 0.6) \). When the random price is such that \( y \in (0.4, 0.6) \), more sellers and buyers join the double auction compared to the case that \( y \in (0, \frac{1}{2}) \). Therefore, buyer BSs are provided with nearer seller BSs on average when \( y \in (0.4, 0.6) \). Thus, the smart grid usage reduces more in the second curve than the first curve of the double-auction-based algorithm. Since in both curves the price is random, the truthfulness of the algorithm is guaranteed. In addition, when \( y \in (0.4, 0.6) \), all bids corresponding to energy portions intended to be sold or to be bought are entered to the double auction. Therefore, the double-auction-based algorithm is iterated once.

The cumulative utilities of a macrocell from serving connected users and energy trades, when the demanded energy profile is common and two different energy harvesting profiles are considered, are shown in Fig. 4 and Fig. 5. In Fig. 4 we compare the cumulative utilities when the matching-game-based algorithm is applied and no algorithm is used. The utility of a BS is found from (7) or (8) in each time slot. The gained profit from serving connected users is \( \psi = 300 \) units of money per consumed Joule. The non-renewable source offers \( \psi = 300 \) units of money per consumed Joule. When the matching-game-based algorithm is not applied, the macrocell gains negative utility values in some time slots due to the high price of the non-renewable energy. To stop gaining negative utility, the macrocell can deploy larger batteries which is an additional cost. Although a higher \( \psi \) can avoid gaining negative utilities, increasing \( \psi \) is out of a BS control. Since the shared energy price per unit is always lower than \( \psi \), and \( \psi \) is more than \( \psi \), negative utility
values can be avoided in time slots by using the matching-game-based algorithm. In Fig. 4, the mean of the harvested energy of a BS is less than the mean of the demanded energy in profile 1, and the mean of the harvested energy of a BS is more than the mean of the demanded energy in profile 2. In both cases, the cumulative utility increases as the matching-game-based algorithm is applied. In the same network with the same energy profiles, the double-auction-based algorithm is applied and the cumulative utility of the same macrocell is given in Fig. 5. It is seen that the cumulative utility of a BS, when each of the proposed algorithms is applied, enhances. Since the extra or needed energy is divided into portions in the double-auction-based algorithm, its computational complexity is more than the matching-game-based algorithm. Therefore, the matching-game-based algorithm performs faster when the size of the network is considerably large.

VI. Conclusion

In this paper, two online energy trading approaches, one decentralized and one centralized, applicable in the small cell networks, are studied where BSs are self-powered. To overcome the uncertainty of the environmental conditions which affects the harvesting energy, two energy trading algorithms are proposed. In the first algorithm, which is decentralized, an energy trading framework is designed by which the BSs with the extra harvested energy are motivated to sell their extra energy to the BSs that have not harvested adequate energy. In addition, BSs with energy deficit are motivated to buy energy from other BSs rather than using the non-renewable energy. This algorithm employs matching theory to assign BSs with energy deficit to BSs with the extra harvested energy. In the second proposed algorithm, which is centralized, different values for energy portions are defined. BSs are entered into a double auction based on their bids. To reduce smart grid usage for transferring energy as well as reducing its usage cost, an optimization is embodied in the proposed algorithm to assigned BSs with energy deficit to near BSs with the extra harvested energy for trading energy. It is demonstrated that the second algorithm satisfies truthfulness, individual rationalities and budget balance. The non-renewable energy consumption is reduced considerably in both algorithms and the cost of installing large batteries is avoided. The non-renewable energy consumption cannot be reduced further as long as the harvested energy and demanded energy are stochastic. Utilizing the proposed algorithms, BSs gain more profit. Therefore, they have incentive to participate in the proposed energy trading scheme. Both of the proposed algorithm are demonstrated to reach Nash equilibrium.

References


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