MIMO Y Channel with Imperfect CSI: Impact of Training and Feedback Overhead

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Abstract—Signal space alignment with network coding (SSA-NC) is a cooperative transmission strategy that, under some conditions such as full channel state information (CSI), provides maximum degrees of freedom (DoF) in two-way relay networks. However, obtaining full CSI for SSA-NC requires a significant amount of training and feedback overhead. This paper focuses on the multiple-input multiple-output (MIMO) Y channel and explicitly considers the required training and feedback at the users and relay. We derive the optimum signaling overhead of this channel to maximize the effective sum-rate of the SSA-NC scheme. We further investigate an important question: under what conditions does the SSA-NC scheme have a better sum-rate performance compared to other popular schemes with lower complexity such as multi-user MIMO (MU-MIMO) and time division multiple access (TDMA)? Since different schemes require different amounts of training/feedback overhead, our result demonstrates how the channel coherence interval plays an important role in their relative performance. Specifically, the SSA-NC scheme outperforms the other schemes only when the coherence interval is sufficiently large. This result highlights the importance of properly taking the training and feedback overhead and channel coherence interval into account when designing the communication scheme for the MIMO Y channel.

Index Terms— MIMO Y channel, signal space alignment, wireless network coding, channel estimation.

I. INTRODUCTION

A. Background and Motivation

In wireless communication systems, cooperative relaying techniques are extensively studied for coverage extension [1]. In this literature, relays cooperate to help the communication from a transmitter to a receiver as one-way relaying scheme or to make a connection between two or more transceivers as two-way relaying scheme. Two-way relay networks (TWRN) have attracted recent attention, thanks to their improved spectral efficiency over one-way relaying systems by making use of bi-directional nature of communication [2]. Specifically, in the generalized version of the TWRN with more than two users in the MIMO fashion, MIMO Y channel is of interest, in which each user transmits its message to the other two users via a relay, while receiving messages from the other two users [3]. Wireless mesh networks, where three nodes are connected by sharing a single relay as tree or star topology, and cellular systems, where a base station is communicated with two users via a relay, are practical examples for MIMO Y channel [4]. In cooperative networks, the information exchange between users with the help of an intermediate relay causes disruptive interference that should be managed. Motivated by operating these networks as close as possible to their optimal performance, many advanced signaling schemes have received attention to handle the interference problem.

B. Our Approach and Contribution

In this paper, we analyze the effect of imperfect CSI obtained through training and analog feedback on the performance of the signal space alignment in MIMO Y channel. Under such a practical situation, it is unclear whether signal space alignment in MIMO Y channel still has the previously reported advantage over the conventional schemes such as TDMA and MU-MIMO. This work aims to provide the answer to this important question, by analyzing the MIMO Y channel using the PNC, taking into account the training and feedback overhead.

We assume that initially all the users and the relay do not have knowledge of CSI. Thus, before data transmission, the wireless channels between the users and the relay need to be trained with the help of analog feedback, in order to enable the signal space alignment. A major contribution of this work is the analytical optimization of the time allocation between the signaling overhead (training and feedback) and data transmission in order to achieve the best sum-rate performance. After finding the optimized time allocation, we further compare the sum-rate performance of the SSA-NC scheme with TDMA and MU-MIMO by deriving the sum-rate performance of these conventional schemes under the same condition (i.e., with optimized signaling overhead). Since different schemes require different amounts of signaling overhead, the channel coherence interval becomes a key determining factor in the relative performance of the schemes. As the channel coherence interval increases from a low value, TDMA provides the highest sum-rate first, then MU-MIMO takes over to provide the best performance, and finally, SSA-NC outperforms the other schemes when the channel coherence interval becomes sufficiently large. Such an interesting performance comparison cannot be revealed if the training and feedback overhead is not taken into account.

II. SYSTEM MODEL

Consider a MIMO Y channel in which three users (K = 3) with M antennas are using an intermediate relay with N
antennas to send two independent messages to the other users in the network. The communication between users via a relay takes two transmission phases, i.e., MAC and BC phases, as depicted in Fig. 1. In the MAC phase, all users simultaneously transmit their signals to the relay. The received signals at the relay in this phase is given by

$$y^r_i = \sum_{i=1}^{K} H^{[r,i]} x^i + n^r_i,$$  

where $x^i \in C^M$ denotes the signal transmitted by user $i$ for $i \in \{1, 2, 3\}$, with the transmit power constraint, i.e., $\mathbb{E}[\text{trace}(x^i_x x^i_x^*)] \leq P$, and accordingly $y^r_i \in C^N$ is the signal received by the relay. The channel linking user $i$ to the relay is denoted by $H^{[r,i]} \in C^{N \times M}$ and $n^r_i$ accounts for complex vector of i.i.d white Gaussian noise at the relay with covariance matrix $\sigma^2 I_N$. In the BC phase, the relay broadcasts the newly generated signals to all users. The received signal at the $i$th user in this phase is given by

$$y^i = H^{[i,r]} x^r + n^i \quad \forall i \in \{1, 2, 3\},$$

where $x^i \in C^N$ is the signal transmitted by the relay with the transmit power constraint, i.e., $\mathbb{E}[\text{trace}(x^i_x x^i_x^*)] \leq P$, and $y^i \in C^M$ defines the signal received by user $i$. In (2), $H^{[i,r]} \in C^{M \times N}$ is channel linking the relay to user $i$ and $n^i$ denotes a vector of i.i.d complex Gaussian noise at the $i$th user with covariance matrix $\sigma^2 I_M$. In order to achieve the maximum DoF in this scheme, relay should equipped with $N \geq \left\{\frac{M}{2}\right\}$ antennas [5], which is assumed to be held in this paper.

Throughout this paper, it is assumed that the entries of the channel matrices are i.i.d as $CN(0, 1)$, and therefore, almost surely, any matrix composed of channel coefficients will have full rank. The channels follow a block-fading model, in which within one transmission phase that takes two time slots, the channels remain unchanged and then change to independent states in the next phase. We model all channels as Rayleigh fading. The signaling overhead for obtaining the required CSI, data transmission in MAC phase and data transmission in

![Fig. 1. MIMO Y channel](image-url)
(ZF) combiner to detect the network coded messages \( w_{r,i} = w_{ij} \oplus w_{jd} \) which gives
\[
(z_{m}^{r,i})^{*} y^{r} = (z_{m}^{r,i})^{*} u_{m}^{r,i} s_{m}^{r,i} + \sum_{(k,l) \neq (m,i)} (z_{m}^{r,i})^{*} u_{k}^{r,i} s_{k}^{r,i} + (z_{m}^{r,i})^{*} n^{r},
\]
where \( u_{m}^{r,i} \) is the \( m \)th vector of matrix \( U^{r,i} \) and linear receivers \( z_{m}^{r,i} \) satisfy the following conditions for perfect signal alignment:
\[
(z_{m}^{r,i})^{*} u_{k}^{r,i} = 0, \quad \forall (k,l) \neq (m,i), \quad \forall i, m.
\]
and \( |(z_{m}^{r,i})^{*} u_{k}^{r,i}| > 0, \quad \forall (k,l) \neq (m,i), \quad \forall i, m. \)

During the BC phase, the relay broadcasts the network coded messages \( w_{r,i}^{r,i} \) using codewords \( s^{r,i} \) along beamforming vectors \( V^{r,i} = \{v_{1}^{r,i}, v_{2}^{r,i}, \ldots, v_{r}^{r,i}\} \) to all users, which is given by
\[
x^{r} = \sum_{i=1}^{K} \sum_{m=1}^{M} v_{m}^{r,i} s_{m}^{r,i}.
\]
The relay designs the beamforming vectors using the network coding based interference nulling beamforming. For example, signal \( s^{r,i} \) contains the message \( w_{r,i}^{r,i} = w_{r,i}^{[2]} \oplus w_{r,i}^{[3]} \) that causes the interference at the receiver of user 1. Thus, the beamforming matrix \( V^{r,i} \) should be in the null space of the channel matrix \( H^{r,i} \), i.e., \( \text{span}(V^{r,i}) \subset \null(H^{r,i}) \). In a similar manner, \( V^{r,j} \) and \( V^{r,k} \) are constructed satisfying \( \text{span}(V^{r,j}) \subset \null(H^{r,j}) \) and \( \text{span}(V^{r,k}) \subset \null(H^{r,k}) \), respectively. Finally, user 1 applies the ZF combiner to detect the signal streams \( s^{r,i} \) and \( s^{r,k} \), so that, it extracts \( w^{[12]} \) and \( w^{[13]} \) as follows
\[
\begin{align*}
\hat{w}^{[12]} &= w^{[r,1]} \oplus w^{[21]} = (w^{[21]} \oplus w^{[12]} \oplus w^{[22]}), \\
\hat{w}^{[13]} &= w^{[r,2]} \oplus w^{[31]} = (w^{[31]} \oplus w^{[13]} \oplus w^{[32]}).
\end{align*}
\]
As a result, by combining of the SSA-NC in the MAC phase and network coding based interference nulling beamforming in the BC phase, the transmission of non-interfering 3M message streams would be possible. The achievable effective sum-rate in this network via Gaussian signaling under full CSI condition is given by
\[
\bar{R}_{eff} = \sum_{i=1}^{K} \sum_{j=1}^{K} \bar{R}_{ij},
\]
where \( \bar{R}_{ij} \) is the achievable data rate from user \( i \) to user \( j \) and \( \mu \in (0, 1) \) denotes the fixed proportion of channel coherence interval \( T \) allocated to the MAC phase as
\[
\bar{R}_{ij} = \text{E} \min \left\{ \frac{1}{\mu} \sum_{m=1}^{M} \log_{2} \left( 1 + \frac{P_{u} E_{r}}{\sigma^{2}} |(z_{m}^{r,i,j})^{*} H^{r,i} v_{m}^{r,i,j}|^{2} \right), \right.
\]
\[
\left. (1 - \mu) \sum_{m=1}^{M} \log_{2} \left( 1 + \frac{P_{u} E_{r}}{\sigma^{2}} |(z_{m}^{r,i,j})^{*} H^{r,i} v_{m}^{r,i,j}|^{2} \right) \right\}.
\]
In this work, we assume that the durations of the MAC and BC phases are equal, i.e., \( \mu = 1 - \mu = 0.5 \).


III. SIGNALING FOR MIMO Y CHANNEL WITH IMPERFECT CSI

During the MAC phase, all users send their messages using the signal space alignment network coding, so that the beamforming vectors in (4) should be determined accurately. Let vectors \( v_{m}^{[21]}, v_{m}^{[12]} \) and \( v_{m}^{[23]} \) are unit vectors designed randomly, and accordingly the vectors \( v_{m}^{[21]}, v_{m}^{[32]} \) and \( v_{m}^{[13]} \) are estimated at users 1, 2 and 3, respectively, providing the relay with aligned desired signals, i.e.,
\[
\begin{align*}
\hat{H}_{1}^{r,i} v_{m}^{[21]} &= \hat{H}_{2}^{r,i} v_{m}^{[12]}, \\
\hat{H}_{3}^{r,i} v_{m}^{[32]} &= \hat{H}_{2}^{r,i} v_{m}^{[13]}, \\
\hat{H}_{2}^{r,i} v_{m}^{[32]} &= \hat{H}_{3}^{r,i} v_{m}^{[23]},
\end{align*}
\]
where \( \hat{H}_{j}^{r,i} \) is estimated channel matrix \( H^{r,i} \) at the user \( j \) with error \( \hat{H}_{j}^{r,i} = H^{r,i} - \hat{H}_{j}^{r,i} \). The received signal model at the relay under the imperfect CSI condition can be represented as
\[
\begin{align*}
y^{r} &= \sum_{i=1}^{K} \left( \hat{H}_{1}^{r,i} V^{r,i} (s^{r,i} + s^{j,i}) \right) + n^{r},
\end{align*}
\]
where \( j = i + 1 \). As can be seen, the first term in the right hand side of (18) contains the aligned signals and the second term is because of the channel estimation error. The ZF combiner at the relay is designed using the information of predetermined beamforming matrices \( V^{r,i} \) and \( V^{r,j} \) and the estimated channels at the relay satisfying
\[
\begin{align*}
(z_{m}^{r,1})^{*} \hat{H}_{2}^{r,2} v_{k}^{[12]} &= 0, \quad |(z_{m}^{r,1})^{*} \hat{H}_{2}^{r,2} v_{m}^{[12]}| > 0, \\
(z_{m}^{r,2})^{*} \hat{H}_{3}^{r,1} v_{k}^{[13]} &= 0, \quad |(z_{m}^{r,2})^{*} \hat{H}_{3}^{r,1} v_{m}^{[13]}| > 0, \\
(z_{m}^{r,3})^{*} \hat{H}_{2}^{r,3} v_{k}^{[23]} &= 0, \quad |(z_{m}^{r,3})^{*} \hat{H}_{2}^{r,3} v_{m}^{[23]}| > 0,
\end{align*}
\]
for \( \forall m, k, m \neq k \). The received signals at the relay is represented as (22), that it is shown at the top of the next page. The first term in the right hand side of (22) contains the aligned signals that should be decoded, while the second and third terms are treated as interference. After detection, the relay designs the beamforming matrices satisfying the following conditions using estimated CSI:
\[
\begin{align*}
\text{span}(V^{r,i}) \subset \null(\hat{H}_{1}^{r,i}), \\
\text{span}(V^{r,j}) \subset \null(\hat{H}_{2}^{r,i}), \\
\text{span}(V^{r,k}) \subset \null(\hat{H}_{3}^{r,i}).
\end{align*}
\]
To investigate the transmission during this phase, we consider the received signals at user 1 and it can be shown that the result is similar to the other users because of the symmetry in the network. The received signal at the receiver of user 1 including
the desired signals $s^{[r,1]}$ and $s^{[r,2]}$ and the interference due to CSI estimation error can be written as

$$
y^{[1]} = \mathbf{H}^{[1r]} [\mathbf{V}^{[r,1]} \mathbf{V}^{[r,2]}] \begin{bmatrix} s^{[r,1]} \\ s^{[r,2]} \end{bmatrix} + \mathbf{H}^{[1r]} \mathbf{V}^{[r,3]} s^{[r,3]} + n^{[1]}.
$$

We define $\mathbf{ES}^{[j,l]}$ as the estimated value of $\mathbf{H}^{[j,r]} \mathbf{V}^{[r,\pi(j,i)]}$ at the user $j$ and accordingly the error of this estimation is denoted by $\mathbf{ER}^{[j,l]}$, where $l = \pi(j,i)$. User 1 applies ZF combiner matrix $\mathbf{Z}^{[l]} = [\mathbf{z}^{[12]} \mathbf{z}^{[13]}]$ using the information of $\mathbf{ES}^{[1,l]}$ for $l \in \{1, 2\}$ such that

$$
(z_{m}^{[12]})^{*} \mathbf{ES}^{[1,1]} = 0, \quad \left| (z_{m}^{[12]})^{*} \mathbf{ES}^{[1,1]} \right| > 0,
$$

$$
(z_{m}^{[13]})^{*} \mathbf{ES}^{[1,2]} = 0, \quad \left| (z_{m}^{[13]})^{*} \mathbf{ES}^{[1,2]} \right| > 0, \quad \forall m \neq k
$$

where $\mathbf{ES}^{[1,l]}$ is the $k$th column of $\mathbf{ES}^{[1,1]}$. The achievable average sum-rate under imperfect CSI condition is given by

$$
\hat{R}_{\text{eff}} = \mu \sum_{k=1}^{2} \sum_{j=1}^{K} \hat{R}_{ji},
$$

where

$$
\hat{R}_{ji} = \min \left\{ \sum_{m=1}^{M} \left( 1 + \frac{P}{M} \left| (z_{m}^{[r,\pi(i,j)]})^{*} \mathbf{H}^{[r]} \mathbf{v}_{m}^{[j]} \right|^{2} \right) I_{ji}^{1} + \sigma^{2} \right\},
$$

$$
\sum_{m=1}^{M} \left( 1 + \frac{P}{N} \left| (z_{m}^{[r,\pi(i,j)]})^{*} \mathbf{ES}^{[j]} \mathbf{v}_{m}^{[j]} \right|^{2} \right) I_{ji}^{2} + \sigma^{2}_{r}
$$

where $I_{ji}^{1}$ is the power of interference produced during the detection of message $w_{ji}$ at the relay, and similarly, $I_{ji}^{2}$ is the power of interference produced during the detection of message $w_{ji}$ at user $j$. Superscript 1 and 2 in $I_{ji}^{1}$ and $I_{ji}^{2}$ denote phase 1 (MAC) and phase 2 (BC), respectively. These interferences in (28) are given by

$$
I_{ji}^{1} = \sum_{i=1}^{K} \sum_{k=1}^{2} \sum_{l=1}^{M} \left( \frac{P}{M} \left| (z_{m}^{[r,\pi(i,j)]})^{*} \mathbf{H}^{[r]} \mathbf{v}_{m}^{[j]} \right|^{2} \right) I_{ji}^{1} + \sigma^{2}
$$

$$
\sum_{m=1}^{M} \left( 1 + \frac{P}{N} \left| (z_{m}^{[r,\pi(i,j)]})^{*} \mathbf{ES}^{[j]} \mathbf{v}_{m}^{[j]} \right|^{2} \right) I_{ji}^{2} + \sigma^{2}_{r}
$$

where $q = l + 1$. We can simplify the expression of equation (29) using the fact that its terms are independent and network is symmetric as,

$$
I_{ji}^{1} = \frac{3MP}{2} (\sigma_{H}^{2} + 2\sigma_{V}^{2}) \quad q = 1, 2, 3.
$$

In addition, the interference expression $I_{ji}^{2}$ can be expressed as

$$
I_{ji}^{2} = \sum_{k=1}^{2} \sum_{q=1}^{3} \left( \frac{P}{N} \left| \right. \sum_{l=1}^{M} (z_{m}^{[j,l]})^{*} \mathbf{ER}^{[j,k,(l,q)]} \left. \right|^{2} \right)
$$

$$
+ (z_{m}^{[j,l]})^{*} \mathbf{H}^{[r]} \mathbf{v}_{m}^{[j]} \right|^{2} \right) I_{ji}^{2} + \sigma^{2}_{r}
$$

Thus, from (31), the simplified form of $I_{ji}^{2}$ can be rewritten as

$$
I_{ji}^{2} = \frac{M^{2}P}{2N} (2\sigma_{H}^{2} + \sigma_{V}^{2} + N\sigma_{V}^{2})
$$

IV. Training and Analog Feedback

The process and time spent on obtaining the CSI is called the signaling overhead. Specifically, it requires training of the channels linking user $i$ to the relay, namely, forward channels, and the channels linking relay to the users, namely, backward channels. The forward channels are trained by the users transmitting pilot symbols to the relay, while the backward channels are trained by the relay sending pilot symbols to the users. Furthermore, all users need to feed back their estimated CSI of the backward channels to the relay. The relay also feeds back the estimated CSI of the forward channels to the users, where both feedback processes are done in an analog fashion. Finally, the relay transmits the information regarding the beamforming matrices $\mathbf{V}^{[r,i]}$ to the users.

In the MAC systems, users send their training signals non-overlapping in time, i.e., $T_{r} = \sum_{k=1}^{K} T_{r,k}$ [7], while the training time of each user $T_{r,k}$ is at least equal to its number of antennas [8]. Thus, for the considered system, at least $KM$ symbols are needed to train the forward channels. In addition to this, the minimum number of symbols for the CSI feedback is $KN$ [9]. In the BC systems, since all users are trained simultaneously, the number of symbols needed to train the channels is at least equal to the number of antennas of relay, i.e., $N$. Moreover, having $KM$-antennas users, the number of CSI feedback symbols should not be less than $KM$ [10].

A. Forward and Backward Channel Training

In the first training phase, similar to [11], each user $i$ sends an orthogonal pilot sequence matrix $\Phi_{i}$, i.e., $\Phi_{i}^{[k]}(\Phi_{i}^{[k]})^{*} =
δ_kI_M to train the forward channels over a training period \( \tau_1 \geq K M \). The relay observes the \( N \times \tau_1 \) matrix \( Y^r \) as
\[
Y^r = \sqrt{\frac{\tau_1 P}{M}} \sum_{i=1}^{K} H_{r,i} \phi[i] + N_r,
\]
where \( N_r \) is an \( N \times \tau_1 \) matrix of Gaussian noise. The MMSE estimate of the channel \( H_{r,i} \) is calculated at the relay as
\[
\hat{H}_{r,i} = \frac{\sqrt{\frac{\tau_1 P}{M}}}{\sigma^2 + \frac{\tau_1 P}{M}} Y^r (\phi[i])^*.
\]
This results the channel estimate as \( \hat{H}_{r,i} \) with i.i.d \( \mathcal{CN}(0, \frac{\tau_1 P}{\sigma^2 + \frac{\tau_1 P}{M}}) \) entries and corresponding error \( \hat{H}_{r,i} - H_{r,i} \) with \( \mathcal{CN}(0, \frac{\tau_1 P}{\sigma^2 + \frac{\tau_1 P}{M}}) \) entries. The training of the backward channels proceeds similarly by the relay. It sends an orthogonal pilot sequence over a training period \( \tau_2 \geq N \). Therefore, the MMSE estimate of channel \( H_{i,r} \) at user \( i \) concludes \( \hat{H}_{i,r} \sim \mathcal{CN}(0, \frac{\tau_3 P}{\sigma^2 + \frac{\tau_3 P}{M N}}) \) with the corresponding error term \( \hat{H}_{i,r} - H_{i,r} \sim \mathcal{CN}(0, \frac{\tau_3 P}{\sigma^2 + \frac{\tau_3 P}{M N}}) \).

### B. Analog Feedback

After training forward and backward channels, all users feed back their channel estimate \( \hat{H}_{i,r} \) in an analog fashion during period \( \tau_3 \). Similar to [12], each user transmits \( M \times \tau_3 \) feedback matrix \( \bar{X}^i \), which utilizes an orthogonal sequence represented by a \( N \times \tau_3 \) matrix \( \Psi^i \) and applies a leading scaling factor to satisfy the transmit power constraints, \( \mathbb{E} (\text{trace}(\bar{X}^i (\bar{X}^i)^*)) = \tau_3 P \), as
\[
\bar{X}^i = \sqrt{\frac{\tau_3 P}{M N (\sigma^2 + \frac{\tau_3 P}{M N})}} \hat{H}_{i,r} \Psi^i, \quad \tau_3 \geq K M.
\]
Thus, the relay can estimate \( \hat{H}_{r,i} \) after receiving feedback symbols by using the information of the channel estimate \( \hat{H}_{r,i} \). An analysis on the feedback estimator and the resulting error have been done in [11]. For brevity, we give the final results in this paper. The entries of the error matrix \( \bar{H}^r_{i,r} \) in this phase are complex Gaussian distributed with the following variance
\[
\sigma^2_{\bar{H}_{r,i}} = \frac{N \sigma^2}{P \tau_2} + \frac{\sigma^2}{(K N - M) P} \left( \frac{M^2}{\tau_1} + \frac{N M}{\tau_3} \right).
\]
The estimated CSI of the forward channel \( H_{r,i} \) at the relay also should be sent to the users over a period \( \tau_3 \geq K N \). Each user needs CSI of the channels \( H_{r,i} \) and \( H_{r,j} \) with \( i \neq j \) in order to design beamforming vectors as stated in Section III. The calculations are similar to the previous feedback phase and because of symmetry in network, the entries of error matrices \( \bar{H}_{i,r} \) and \( \bar{H}_{r,i} \) have equal variance as follows
\[
\sigma^2_{\bar{H}_{r,i}} = \frac{M \sigma^2}{\tau_1 P} + \frac{\sigma^2}{(K M - N) P} \left( \frac{N^2}{\tau_2} + \frac{K N M}{\tau_4} \right).
\]

### C. Transmitting the Information of Beamforming Vectors of the Relay

As stated in Section III, each user \( i \) needs the knowledge of the beamforming vectors of the relay (more precisely it needs the knowledge of the product of the relevant backward channel matrix and the corresponding beamforming matrix used by the relay) for detection, and thus, another period of duration \( \tau_5 \) is allocated for obtaining relay’s beamforming vector at the users. In this phase, the relay post-multiplies its \( N \times N \) beamforming matrix \( V^r = [V^r_{r,1}, V^r_{r,2}, V^r_{r,3}] \) with a \( N \times \tau_5 \) matrix \( \Lambda = [\Lambda_{r,1}, \Lambda_{r,2}, \Lambda_{r,3}] \) such that \( \Lambda^T \hat{H}_{r,i} \Lambda^* = \delta_{i,k} \frac{1}{2} \).
The transmitted \( N \times \tau_5 \) matrix \( \bar{X}^r \) from the relay in this phase is given by
\[
\bar{X}^r = \sqrt{\frac{\tau_5 P}{N}} [V^r_{r,1}, V^r_{r,2}, V^r_{r,3}] \left[ \Lambda^T_{r,1} \Lambda_{r,2} \Lambda_{r,3} \right].
\]
As a result, user \( 1 \) post-multiplies the received symbols \( \bar{Y}^i \) by \( \Lambda^T_{r,3} \) to compute
\[
\bar{Y}^i \Lambda^T_{r,3} = \sqrt{\frac{\tau_5 P}{N}} \bar{X}^r V^r_{r,1} V^r_{r,2} \left[ \Lambda^T_{r,1} \Lambda_{r,2} \right] + N^i [\Lambda^T_{r,3}]^*.
\]
Finally, user \( 1 \) estimates the value of \( (H^r_{1r} V^r_{r,1}) \) and \( (H^r_{1r} V^r_{r,2}) \), while the entries of the corresponding error matrices have i.i.d distributions with equal variance, i.e., \( \mathcal{CN}(0, \frac{\sigma^2}{\tau_5 P}) \).

### V. PERFORMANCE COMPARISON WITH MU-MIMO AND TDMA SCHEMES

In order to compare the performance of the SSA-NC scheme with the conventional schemes with less complexity, including TDMA and MU-MIMO, we further study the signaling overhead of TDMA and MU-MIMO schemes in the next section. Then, we numerically compare the three schemes to have a improved understanding on their relative performance advantage in the practical case of imperfect CSI.

#### A. Data Transmission of MU-MIMO Scheme

In MIMO Y channel, exchanging messages between the users via a relay requires four orthogonal time slots in the MU-MIMO scheme [14]. Without loss of generality, we analyze this scheme in first and second time slots, where message vectors \( w^{[1]} \), \( w^{[3]} \) and \( w^{[4]} \) are sent. The transmission of message vectors \( w^{[2]} \) and \( w^{[3]} \) in the third and fourth time slot can be analyzed similarly.

During the MAC phase, the users send their messages using random beamforming vectors, or based on a pre-specified decision mechanism. The relay requires \( N = \frac{3}{2} M \) antennas to decode the received messages successfully. Then, the beamforming matrices are designed at the relay using network coding based interference nulling beamforming. If two first time slots take \( T \) symbols, a fraction of these symbols should be used for the channel training as shown in Fig. 2.
The channel training in this scheme is divided into four phases. The training of forward channel and backward channel each takes $\tau_1 \geq KM$ and $\tau_2 \geq N$ training periods, respectively. Both the feedback of backward channel estimations and relay beamforming vectors information take $\tau_3 \geq KN$ and $\tau_4 \geq N$ training period, respectively.

Similar to the SSA-NC scheme, optimization of the effective sum-rate concludes the optimum training periods as follows

$$
\begin{align*}
\tau_3 &= \frac{\alpha T}{1 + \beta_1 + \beta_2 + \beta_4}, \\
\tau_1 &= \beta_1 \tau_3, \\
\tau_2 &= \beta_2 \tau_3, \\
\tau_4 &= \beta_4 \tau_3,
\end{align*}
$$

where

$$
\begin{align*}
\beta_1 &= \sqrt{\frac{3P(KN - M)}{2NP_r} + \frac{M}{N}}, \\
\beta_2 &= \sqrt{\frac{P(KN - M)}{MP_r}}, \\
\beta_4 &= \sqrt{\frac{P(2N - M)}{2NMP_r}}.
\end{align*}
$$

B. Data Transmission of TDMA Scheme

TDMA is another conventional scheme for exchanging messages between the users via a relay in the MIMO Y channel. This scheme requires 6 time slots [14]. In this section, we analyze this scheme in the first and second time slots, and the transmission in other time slots can be investigated similarly. In the first time slot, message vectors $\mathbf{w}_{1}^{[23]}$ and $\mathbf{w}_{3}^{[31]}$ are sent from user 1 using random beamforming vectors, or based on a pre-specified decision mechanism. During the second time slot, the relay uses network coding based interference nulling beamforming to send the decoded messages. In this scheme, if first two time slots take $T$ symbols, the signaling overhead is adopted as shown in Fig. 2. Similar to the MU-MIMO scheme, the channel training in this scheme is divided into four phases, i.e., forward channel training, backward channel training, feedback of backward channel estimation to the relay and the feedback of relay beamforming vectors information over $\tau_1 \geq M$, $\tau_2 \geq N$, $\tau_3 \geq 2N$ and $\tau_4 \geq N$ periods, respectively. Finally, optimization of the effective sum-rate concludes the optimum training periods as follows

$$
\begin{align*}
\tau_3 &= \frac{\alpha T}{1 + \beta_1 + \beta_2 + \beta_4}, \\
\tau_1 &= \beta_1 \tau_3, \\
\tau_2 &= \beta_2 \tau_3, \\
\tau_4 &= \beta_4 \tau_3,
\end{align*}
$$

where

$$
\begin{align*}
\beta_1 &= \sqrt{\frac{P(2N - M)}{NP_r} + \frac{M}{N}}, \\
\beta_2 &= \sqrt{\frac{P(2N - M)}{MP_r}}, \\
\beta_4 &= \sqrt{\frac{P(KN - M)}{NMP_r}}.
\end{align*}
$$

C. Performance Analysis of Three Schemes

Now, we compare the sum-rate performance of the three schemes, as shown in Fig. 3. As mentioned previously, the TDMA scheme requires less training length than the other schemes since less channels should be trained in the training phase. This leads to more symbols allocated to the data transmission for reaching the good quality estimated channels. Thus, the TDMA scheme achieves a higher sum-rate in low range of the channel coherence interval $T$. Specifically, since more channels should be trained, the MU-MIMO and SSA-NC schemes result in lower sum-rate in low ranges of channel coherence interval $T$.

As the channel coherence interval $T$ increases, MU-MIMO scheme overcomes the TDMA scheme because more data are sent during the data transmission phase. Furthermore, comparing to SSA-NC scheme, this scheme achieves more sum-rate, because of requiring less training phase length. Hence, in the average range of channel coherence interval $T$, Fig. 3. Effective sum-rate in MIMO Y channel versus channel coherence interval $T$ for SSA-NC, MU-MIMO and TDMA with $P = P_r = 10dB$ and $\sigma^2 = 0dB$. The parameter $T_{t,max}$ is the maximum of the minimum required channel coherence interval $T$ for three schemes.

Fig. 4. Effective sum-rate in MIMO Y channel versus power of relay and users for SSA-NC scheme, the MU-MIMO and TDMA with $T = 100$ and $\sigma^2 = 0dB$. 

the MU-MIMO scheme has the best performance. Finally, in the high values of channel coherence interval $T$, the SSA-NC scheme outperforms the other schemes when there are enough symbols for training and also the data transmission.

In what follows, we consider the effect of the power of relay and users on the achievable sum-rate for three schemes. As power of relay and users increases, the achievable sum-rate increases for three schemes. We consider two different channel coherence interval, e.g., $T = 50$ and $T = 100$. In particular, the case of $T = 50$ should be interesting since the sum-rate for all three schemes are very close to each other according to Fig. 3. For the case of $T = 100$, we expect that the SSA-NC performs the best. First, we present the case of $T = 100$ in Fig. 4. The general trend is that the SSA-NC outperforms the other two schemes and the relative advantage increases as the power goes up. Nevertheless, the relative advantage of SSA-NC is insignificant when power is low, which implies that the SSA-NC should not be used with low power budget due to its complexity.

Fig. 5 shows the effective sum-rate versus power for less coherence interval ($T = 50$). As shown in this figure, the TDMA scheme gets the best sum-rate for the power below 10 dB. As power increases, the MU-MIMO outperforms the TDMA for the maximum power of 21.5 dB. Finally, the SSA-NC scheme achieves the highest sum-rate for the high power ranges in comparison to the others.

REFERENCES